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# Asset Pricing with Spatial Interaction

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We propose a spatial capital asset pricing model (S-CAPM) and a spatial arbitrage pricing theory (S-APT) that extend the classical asset pricing models by incorporating spatial interaction. We then apply the S-APT to study the comovements of eurozone stock indices (by extending the Fama-French factor model to regional stock indices) and the futures contracts on S&P/Case-Shiller Home Price Indices; in both cases spatial interaction is significant and plays an important role in explaining cross-sectional correlation.

*Key words*: capital asset pricing model, arbitrage pricing theory, spatial interaction, real estate, factor model, property derivatives, futures
*History*:

## 1. Introduction

A central issue in financial economics is to understand the risk-return relationship for financial assets, as exemplified by the classical capital asset pricing model (CAPM) and arbitrage pricing theory (APT). Building on the seminal work of Markowitz (1952), CAPM, as proposed by Sharpe (1964) and others, characterizes the market equilibrium when all market participants hold mean-variance efficient portfolios. Unlike the CAPM, the APT introduced by Ross (1976a,b) is based on an asymptotic arbitrage argument rather than on market equilibrium, which allows for multiple risk factors and does not require the identification of the market portfolio; see e.g., Huberman (1982), Chamberlain (1983), Chamberlain and Rothschild (1983), Ingersoll (1984), Huberman and Wang (2008), among others.

In terms of empirical performance, APT improves on CAPM in that cross-sectional differences in expected asset returns are better accounted for by multiple factors in APT; see the Nobel-prize-winning work of Fama and French (1993) and Fama and French (2012), among others. Going beyond expected returns, however, the APT models with the famous factors in existing literature do not seem to capture all the cross-sectional variations in realized asset returns. In particular, a motivating example of fitting an APT model to the European countries stock indices returns (see Section 2.1) shows that (i) there is evidence of cross-sectional spatial interaction among the residuals of the APT regression model; (ii) the no asymptotic arbitrage constraint (i.e., zero-intercept constraint) implied by the APT is rejected by the data, which demonstrates that the existing APT model and famous factors are not adequate in accounting for the cross-sectional variations.

To better account for potential spatial correlation among residuals of APT models and to better capture the no asymptotic arbitrage constraint, in this paper, we attempt to link spatial econometrics, which emphasizes the statistical modeling of spatial interaction, with the classical CAPM and APT. Empirical importance of spatial interaction has already been found in the real estate markets (see, e.g., Anselin (1988) and Cressie (1993)), in U.S. equity market (see Pirinsky and Wang (2006) for comovements of common stock returns of US corporations in the same geographic area), and in international stock portfolios (see Bekaert, Hodrick, and Zhang (2009)). Coval and Moskowitz (2001) demonstrated empirically the importance of spatial information in the investment decisions and outcomes of individual fund managers.

In this paper we study the impact of spatial information on overall markets in the form of CAPM or APT. More precisely, we first propose a spatial capital asset pricing model (S-CAPM) and a spatial arbitrage pricing theory (S-APT), and then study empirical implications of the models. The new models can be applied to financial assets that can be sold short, such as national/regional stock indices and futures contracts on the S&P/Case-Shiller Home Price Indices (Case and Shiller (1987)).

Our S-CAPM and S-APT differ from existing models in spatial econometrics. The consideration of equilibrium pricing and no arbitrage pricing imposes certain constraints on the parameters in the S-CAPM and S-APT models (see Eq. (15) and Eq. (26) in Theorem 2); these constraints are the manifestation of both the effect of spatial interaction and the economic rationale of asset pricing. In contrast, the parameters in existing spatial econometric models are generally not subject to constraints.

After developing the economic models, we give two applications of the proposed S-APT. First, we continue the investigation of the motivating example in Section 2.1, in which the comovements of returns of eurozone stock indices are studied, by extending the factor model for international stocks proposed in Fama and French (2012) to incorporate spatial interaction. Factor models for stock markets have been well studied in the literature. In the ground-breaking work of Fama and French (1993), two factors related to firm size and book-to-market equity are constructed and shown to have great explanatory power of cross-sectional stock returns. In their approach, a factor is constructed as the difference between the returns of firms with certain characteristics (e.g. small-cap) and those with opposite characteristics (e.g. large-cap). This approach has at least two advantages. First, factors constructed in this way are payoffs of zero-cost portfolios that are traded in the market and further steps of linear projection of factors are unnecessary. Second, the factors have clear economic interpretation. For instance, since the book-to-market ratio is indicative of financial distress, the factor constructed according to the ratio can be viewed as a proxy for distress risk. Fama and French (2012) investigate the performance of the market, size, value, and momentum factors in international stock markets.

We extend Fama and French (2012) in three ways: (i) By using the S-APT model instead of APT models (factor models without spatial interaction), we investigate the role of spatial interaction in the explanation of comovements of stock index returns. We find that spatial interaction is significant, even after controlling for popular factors, including market, size, value, and momentum factor. (ii) Adding spatial interaction for the comovements not only improves the overall model fitting in terms of Akaike information criterion (AIC), but also reduces the degree of spatial correlation (i.e., $\kappa$ in Eq. (2)) among residuals of the fitted model. Furthermore, the no asymptotic arbitrage constraint (i.e., the zero-intercept constraint) is no longer rejected by the data after spatial interaction is incorporated in the model. (iii) We focus on eurozone stock indices that are portfolios of stocks implicitly sorted by locations/nations, while Fama and French (2012) study the returns of stock portfolios constructed according to other issuer characteristics such as size and value.

As the second application, we apply the S-APT model to the study of risk-return relationship of real estate securities, particularly the S&P/Case-Shiller Home Price Indices (CSI Indices) futures. The CSI Indices are constructed based on the method proposed by Case and Shiller (1987) and are the leading measure of single family home prices in the United States. It is important to study the risk-return relationship of real estate securities such as the CSI Indices futures because they are useful instruments for risk management and for hedging in residential housing markets (Shiller (1993)), similar to the function that futures contracts fulfill in other financial markets; see Fabozzi, Shiller, and Tunaru (2012) and the references therein for the pricing and use of property derivatives for risk management.

We add to the literature on the study of real estate securities by constructing a three-factor S-APT model for the CSI Indices futures returns. Using monthly return data, we find that the spatial interaction among CSI indices futures returns are significantly positive. In addition, the no asymptotic arbitrage (i.e., zero-intercept) constraint implied by the S-APT model is not rejected by the futures data. Furthermore, incorporating spatial interaction improves the model fitting to the data in terms of AIC and eliminates the spatial correlation among the residuals of APT models.

Our paper significantly differs from existing literature that also incorporates spatial information. For example, our paper differs from Ortalo-Magné and Prat (2013) (OP) mainly in the following aspects: (i) Our paper focuses on the S-APT and its implications; OP does not study S-APT (or asset pricing) under the no asymptotic arbitrage assumption. (ii) In our S-CAPM model, we assume that the spatial interaction between the returns of assets is exogenously given as in (6); OP does not make such an assumption. Instead, OP considers a general equilibrium model where the city-specific productivity of agents, city-specific income of agents, and stock dividends are exogenously given, but agent's choice of where to live, stock prices, city-specific housing prices, and city-specific rent prices and their returns are determined by the general equilibrium. (iii) Our paper concerns the risk and return of real estate securities that are liquid and can be easily shorted, such as futures contracts on the CSI Indices; OP derives the equilibrium prices of houses and rents that are illiquid and difficult to be sold short. (iv) Our S-CAPM model assumes that investors hold mean-variance efficient portfolios, but OP assumes that the utility function of agents is the CARA utility. (v) In our S-CAPM model, the market portfolio is still the same as the classical CAPM, which is the value weighted portfolio of all assets; however, in the CAPM derived under the model of OP, the real estate in the market portfolio is adjusted by the hedging demand of agents. Our paper also differs from Fernandez (2011) in that: (i) Our paper focuses on the S-APT; Fernandez (2011) does not study S-APT (or asset pricing) under the no asymptotic arbitrage assumption. (ii) We rigorously proves that Eq. (15) of this paper holds under the assumption in Theorem 1, but Fernandez (2011) just *assumes* that Eq. (15) is true without any theoretical justification. (iii) We derive the S-CAPM in Theorem 1, which is not obtained in Fernandez (2011). (iv) Our S-CAPM incorporate futures returns which are not considered in Fernandez (2011).

In summary, the main contribution of this paper is twofold: (i) Theoretically, we extend the classical asset pricing theories of CAPM and APT by proposing a spatial CAPM (S-CAPM) and a spatial APT (S-APT) that incorporate spatial interaction. The S-CAPM and S-APT characterize how spatial interaction affects asset returns by assuming, respectively, that investors hold mean-variance efficient portfolios and that there is no asymptotic arbitrage. In addition, we develop estimation and testing procedures for implementing the S-APT model. (ii) Empirically, we apply the S-APT models to the study of the eurozone stock indices returns and the futures contracts written on the CSI Indices. In both cases, the spatial interaction incorporated in the S-APT model seems to be a significant factor in explaining asset return comovements.

The remainder of the paper is organized as follows. In Section 2, a motivating example is discussed and a linear model with spatial interaction is introduced. The S-CAPM and S-APT for ordinary assets and futures contracts are derived in Sections 3 and 4, respectively. Section 5 develops the econometric tools for implementing the S-APT model. The rigorous econometric analysis of the identification and statistical inference problems for the proposed spatial econometric model is given in E-Companion EC.4. The empirical studies on the eurozone stock indices and the CSI Indices futures using the S-APT are provided in Sections 6 and 7, respectively. Section 8 concludes.

## 2. Preliminary

### 2.1. A Motivating Example of Spatial Correlation

We consider the comovements of the returns of stock indices in developed markets in the European region. To minimize the effect of exchange rate risk, we restrict the study in the eurozone, which consists of the countries that adopt Euro as their currency. In total, there are 11 countries with developed stock markets in the eurozone. The data consists of the monthly simple returns of stock indices of these countries; see Table 1. Like Fama and French (2012), all returns are converted and denominated in US dollar. Since Greece adopted the Euro in the year 2000, the time period of the data spans from January 2001 to October 2013. Since all returns are denominated in US dollar, the simple return of one-month US treasury bill is used as the risk-free return.

| Country | Austria | Belgium | Finland | France | Germany | Greece | Ireland | Italy | Netherlands | Portugal | Spain |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Stock Index | ATX | BEL20 | HEX | CAC | DAX | ASE | ISEQ | FTSEMIB | AEX | BVLX | IBEX |

**Table 1 The stock indices of the 11 eurozone countries with developed stock markets.**

We apply the following APT model to the monthly excess returns of the 11 stock indices:

$$r_{it} - r_{ft} = \alpha_i + \sum_{k=1}^{4} \beta_{ik} f_{kt} + \epsilon_{it}, i = 1, \dots, 11; t = 1, \dots, T; \tag{1}$$

where $r_{it}$ is the return of the $i$th stock index in the $t$th month; $r_{ft}$ is the risk-free return in the $t$th month; $f_{kt}, k = 1, 2, 3, 4$, are respectively the market, size, value, and momentum factors in the $t$th month. The four factors are defined in Fama and French (2012), and the data for the four factors are downloaded from the website of Kenneth R. French.[1] $\beta_{ik}$ is the factor loading of the $i$th stock

[1] These factors are called the "Fama/French European Factors" which can be downloaded at `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/Europe_Factors.zip`. These factors are constructed from the stocks in the developed European countries including Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. All these factors are based on U.S. dollar denominated stock returns. The detailed description of these factors can be found at `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_developed.html`.

index excess return on the $k$th factor. $\epsilon_{it}$ is the residual. To investigate potential spatial correlation among the 11 return residuals, we consider the following model for $\tilde{\epsilon}_t = (\epsilon_{1t}, \epsilon_{2t}, \ldots, \epsilon_{11t})'$:

$$\tilde{\epsilon}_t = \kappa W \tilde{\epsilon}_t + a + \tilde{\xi}_t, t = 1, 2, \ldots, T, \tag{2}$$

where $W = (w_{ij})$ is a $11 \times 11$ matrix defined as $w_{ij} := (s_i d_{ij})^{-1}$ for $i \neq j$ and $w_{ii} = 0$, where $d_{ij}$ is the driving distance between the capital of country $i$ and that of country $j$ and $s_i := \sum_j d_{ij}^{-1}$; $\kappa$ is a scalar parameter; $a$ is a vector of free parameters; $\tilde{\xi}_t$ is assumed to have a normal distribution $N(0, \sigma^2 I_{11})$ with $\sigma$ being an unknown parameter and $I_{11}$ being the $11 \times 11$ identity matrix. When $\kappa$ is not zero, each component of $\tilde{\epsilon}_t$ is influenced by other components to a degree dependent on their spatial distances.[2]

We fit the model (2) to the residuals and found that the spatial parameter $\kappa$ for the residuals is statistically positive with a 95% confidence interval of $[0.02, 0.21]$.[3] This provides evidence that there is statistically significant spatial correlation among the residuals that is not adequately captured by the four factors in the APT model.

In addition, we carry out the following hypothesis test of the no asymptotic arbitrage constraint (i.e., zero-intercept constraint) of the APT model:

$$H_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_{11} = 0; \;\; H_1 : \text{else}. \tag{3}$$

We can test the hypothesis using the conditional likelihood ratio test statistic:

$$LR = 2\left[\sum_{t=1}^{T} l(\tilde{r}_t \mid \tilde{f}_t, \hat{\theta}) - \sum_{t=1}^{T} l(\tilde{r}_t \mid \tilde{f}_t, \theta^*)\right],$$

where $\tilde{r}_t = (r_{1t}, r_{2t}, \ldots, r_{11t})$ and $\tilde{f}_t = (f_{1t}, \ldots, f_{4t})$. $\sum_{t=1}^{T} l(\tilde{r}_t \mid \tilde{f}_t, \hat{\theta})$ denotes the conditional log likelihood function evaluated at $\hat{\theta}$, which is the conditional MLE of parameters estimated with no constraints; while $\sum_{t=1}^{T} l(\tilde{r}_t \mid \tilde{f}_t, \theta^*)$ its counterpart evaluated at the conditional MLE $\theta^*$ estimated under the constraint that the null holds (i.e., $\alpha_i = 0$, for all $i$). Under the null hypothesis, the conditional likelihood ratio test statistic has an asymptotic $\chi^2(11)$ distribution; see Section 5.3 for more discussion on the distribution of the test statistic. We find that

$$\text{The p-value of the test (3)} = 0, \tag{4}$$

which provides further evidence that the four factors may not capture the comovements of the indices returns well enough. The inadequacy of the APT model (1) for explaining the comovements of the indices returns is summarized in Table 2.

[2] Using the term $\kappa W \tilde{\epsilon}_t$ to incorporate spatial interaction is proposed in Whittle (1954) and has been widely used in spatial statistics and spatial econometrics (see, e.g., Ord (1975), Cressie (1993), and Lesage and Pace (2009)).

[3] The estimate and confidence interval for $\kappa$ can be obtained by letting $K = 0$ in Eq. (36)-(39) and Eq. (EC.79)-(EC.80) in the E-Companion.

| Questions | Answers from the data |
|---|---|
| Do the residuals in (1) have spatial correlation? | Yes, because the 95% confidence interval of $\kappa$ in (2) is $[0.02, 0.21]$. |
| Is the no asymptotic arbitrage (zero-intercept) test (3) rejected by the data? | Yes, because of (4). |

**Table 2 The inadequacy of the APT model (1) for explaining the comovements of the 11 eurozone national stock indices excess returns.**

It is probable that the aforementioned unsatisfactory performance of the APT model is due to misspecification of factors. To explore this possibility, we run the Ramsey Regression Equation Specification Error Test (RESET), one of the most popular specification test for linear regression models. In our application, RESET tests whether non-linear combinations of current factors have any power in explaining the excess returns on the left-hand side of the APT regression. The intuition behind the test is that if non-linear combinations of factors have any power in explaining cross-sectional excess returns, then the factors of the APT model is misspecified. For a detailed and technical discussion of the RESET, see Ramsey (1969).

The RESET finds weak evidence of factor misspecification. Indeed, RESET indicates that among the eleven excess returns of the national stock indices, only three may benefit from additional factors. Moreover, it is not clear whether the additional factor(s) can help explain the spatial correlation observed in the residuals. Our S-APT model, to be presented in the rest of the paper, provides a unified way to address the spatial correlation in residuals. Furthermore, the no asymptotic arbitrage (i.e., zero-intercept) constraint is not rejected by the data under our S-APT model.

### 2.2. A Model of Spatial Interaction

Consider a one-period economy with $n$ risky assets in the market whose returns are governed by the following linear model:

$$r_i = \rho \sum_{j=1}^{n} w_{ij} r_j + \alpha_i + \epsilon_i, \; i = 1, \ldots, n, \tag{5}$$

where $r_i$ is the uncertain return of asset $i$, $\alpha_i$ is a constant, and $\epsilon_i$ is the residual noise related to asset $i$. For $i \neq j$, $w_{ij}$ specifies the influence of the return of asset $j$ on that of asset $i$ due to spatial interaction; and $w_{ii} = 0$. The degree of spatial interaction is represented by the parameter $\rho$. Let $\tilde{r} := (r_1, \ldots, r_n)'$, $W := (w_{ij})$, $\alpha := (\alpha_1, \ldots, \alpha_n)'$, and $\tilde{\epsilon} := (\epsilon_1, \ldots, \epsilon_n)'$. Then, the above model can be represented as

$$\tilde{r} = \rho W \tilde{r} + \alpha + \tilde{\epsilon}, \;\; E[\tilde{\epsilon}] = 0, \;\; E[\tilde{\epsilon}\tilde{\epsilon}'] = V. \tag{6}$$

Following the convention in spatial econometrics, we assume that the spatial weight matrix $W$ is exogenously given. $W$ is typically defined using quantities related to the location of assets, such as distance, contiguity, and relative length of common borders. For instance, $W$ can be specified as

$w_{ii} = 0$ and $w_{ij} = d_{ij}^{-1}$ for $i \neq j$, where $d_{ij}$ is the distance between asset $i$ and asset $j$. If other asset returns do not have spatial influence on $r_i$, then the $i$th row of $W$ can simply be set to zero.

Henceforth, we assume that $\rho^{-1}$ is not an eigenvalue of $W$. Then, $I_n - \rho W$ is invertible[4] and (6) can be rewritten as

$$\tilde{r} = (I_n - \rho W)^{-1}\alpha + (I_n - \rho W)^{-1}\tilde{\epsilon}, \tag{7}$$

where $I_n$ is the $n \times n$ identity matrix. The mean and covariance matrix of $\tilde{r}$ are thus given by

$$\mu = E[\tilde{r}] = (I_n - \rho W)^{-1}\alpha, \;\; \Sigma = Cov(\tilde{r}) = (I_n - \rho W)^{-1} V (I_n - \rho W')^{-1}. \tag{8}$$

## 3. The Spatial Capital Asset Pricing Model

In this section we develop a spatial capital asset pricing model (S-CAPM) that generalizes the CAPM by incorporating spatial interaction. In our study, it is important to consider futures contracts as stand-alone securities rather than as derivatives of the underlying instruments because the instruments underlying futures contracts in the real estate markets may not be tradable. For example, the CSI Indices futures are traded at Chicago Mercantile Exchange but the underlying CSI Indices cannot be traded directly.

Therefore, we develop the S-CAPM for both ordinary assets and futures contracts. More specifically, suppose in the market there are $n_1$ ordinary risky assets with returns $(r_1, \ldots, r_{n_1})$, a risk-free asset with return $r$, and $n_2$ futures contracts. The return of a futures contract cannot be defined in the same way as that of an ordinary asset because the initial value of a futures contract is zero. Hence, we follow the convention in the literature (see, e.g., De Roon, Nijman, and Veld (2000)) and define

$$r_{n_1+i} := \frac{F_{i,1} - F_{i,0}}{F_{i,0}} \tag{9}$$

as the "nominal return" of the $i$th futures contract, where $F_{i,0}$ and $F_{i,1}$ are the futures prices of the $i$th futures contract at time 0 and time 1 (the beginning and end of the trading period), respectively, and $i = 1, \ldots, n_2$. Let $n = n_1 + n_2$ and assume that the $n$ returns $\tilde{r} = (r_1, \ldots, r_{n_1}, r_{n_1+1}, \ldots, r_n)'$ satisfy the model (6). Then, the mean $\mu$ and covariance matrix $\Sigma$ of $\tilde{r}$ are given by (8).

Now consider the mean-variance problem faced by an investor who can invest in the $n_1$ ordinary assets and $n_2$ futures contracts. Because the investor's portfolio includes both ordinary assets and futures contracts, the return of the portfolio has to be calculated more carefully than if there were no futures contracts in the portfolio. Then, the mean-variance analysis can be carried out; see E-Companion EC.1. Because both $\mu$ and $\Sigma$ are functions of $\rho$ and $W$, the optimal portfolio

[4] Let $\det(\cdot)$ denote matrix determinant and $\omega_1, \ldots, \omega_n$ be the eigenvalues of $W$. Then, $\det(I_n - \rho W) = \prod_{j=1}^{n}(1 - \rho\omega_j) \neq 0$ if and only if $\rho^{-1}$ is not an eigenvalue of $W$.

weights obtained by the mean-variance analysis and the efficient frontiers are affected by spatial interaction. For example, Figure 1 shows the efficient frontiers for different values of $\rho$ with all the other parameters in the model (6) fixed for a portfolio of $n_1 = 10$ ordinary assets and $n_2 = 0$ futures contracts. It is clear that the efficient frontiers are significantly affected by $\rho$. The parameters $W$, $\alpha$, and $V$ used in calculating the efficient frontiers in Figure 1 are randomly generated to have the following values:

$$W = \begin{pmatrix} 0 & 0.080 & 0.131 & 0.206 & 0.054 & 0.055 & 0.128 & 0.068 & 0.204 & 0.075 \\ 0.119 & 0 & 0.082 & 0.193 & 0.067 & 0.055 & 0.223 & 0.069 & 0.103 & 0.089 \\ 0.129 & 0.055 & 0 & 0.086 & 0.073 & 0.093 & 0.066 & 0.122 & 0.273 & 0.103 \\ 0.197 & 0.125 & 0.083 & 0 & 0.048 & 0.044 & 0.261 & 0.056 & 0.119 & 0.068 \\ 0.070 & 0.059 & 0.097 & 0.066 & 0 & 0.119 & 0.056 & 0.212 & 0.093 & 0.228 \\ 0.081 & 0.055 & 0.139 & 0.068 & 0.135 & 0 & 0.058 & 0.235 & 0.107 & 0.122 \\ 0.143 & 0.169 & 0.075 & 0.306 & 0.048 & 0.044 & 0 & 0.054 & 0.096 & 0.064 \\ 0.072 & 0.050 & 0.132 & 0.062 & 0.173 & 0.169 & 0.051 & 0 & 0.108 & 0.181 \\ 0.183 & 0.062 & 0.248 & 0.111 & 0.064 & 0.065 & 0.077 & 0.091 & 0 & 0.099 \\ 0.082 & 0.066 & 0.115 & 0.078 & 0.194 & 0.091 & 0.063 & 0.188 & 0.122 & 0 \end{pmatrix},$$

$\alpha = (1.334\%, 1.005\%, 1.209\%, 1.141\%, 1.101\%, 1.352\%, 3.531\%, 8.229\%, 1.101\%, 1.893\%)'$, and $V = 0.015 \cdot I_{10}$ where $I_{10}$ is a $10 \times 10$ identity matrix.

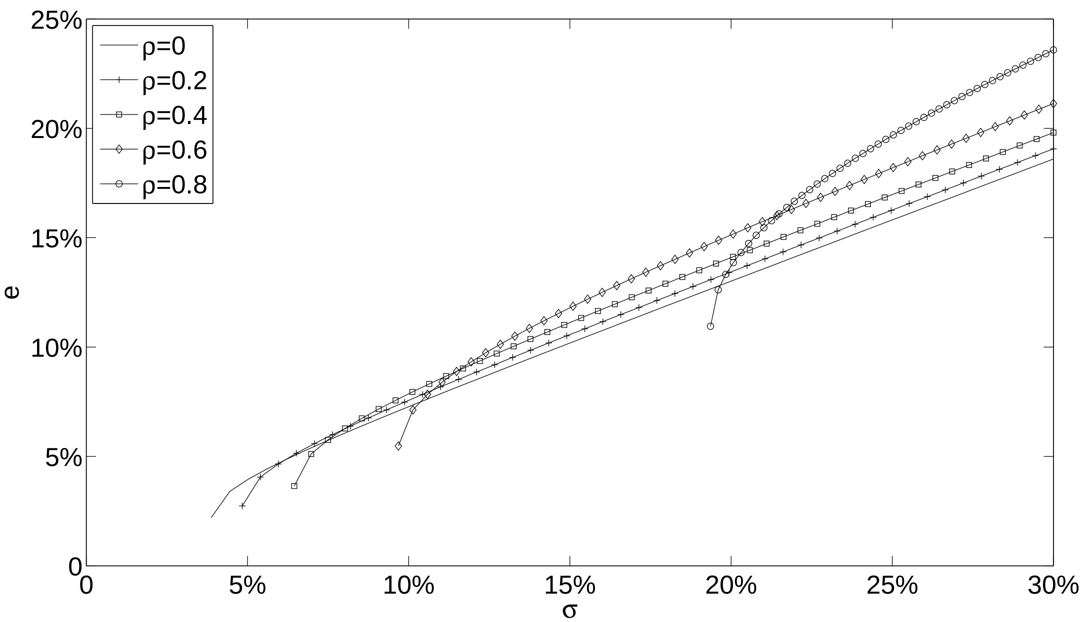



**Figure 1** **Efficient frontiers for $\rho = 0, 0.2, 0.4, 0.6$, and $0.8$, respectively, when there is no risk-free asset. $W$, $\alpha$, and $V$ are specified above. The efficient frontiers are significantly affected by $\rho$.**

Based on the mean-variance analysis, we derive the following S-CAPM, which characterizes how spatial interaction affects expected asset return under market equilibrium.

THEOREM 1. *(S-CAPM for Both Ordinary Assets and Futures) Suppose that there exists a risk-free return $r$ and that the $n = n_1 + n_2$ risky returns satisfy the model (6), of which the first $n_1$ are returns of ordinary assets and the others are returns of futures contracts. Suppose $n_1 > 0$.*[5] *Let $r_M$ be the return of market portfolio. If each investor holds a mean-variance efficient portfolio, then, in equilibrium, $r_M$ is mean-variance efficient and every investor holds only the market portfolio and the risk-free asset. Furthermore,*

*(i) for the ordinary assets,*

$$E[r_i] - r = \frac{Cov(r_i, r_M)}{Var(r_M)}(E[r_M] - r) = \frac{\phi_M' \Sigma \eta_i}{\phi_M' \Sigma \phi_M}(E[r_M] - r), \ i = 1, \ldots, n_1; \tag{10}$$

*(ii) for the futures contracts,*

$$\begin{aligned} E[F_{i,1}] - F_{i,0} &= \frac{Cov(F_{i,1}, r_M)}{Var(r_M)}(E[r_M] - r) \\ &= F_{i,0}\frac{\phi_M' \Sigma \eta_{n_1+i}}{\phi_M' \Sigma \phi_M}(E[r_M] - r), \ i = 1, \ldots, n_2, \end{aligned} \tag{11}$$

*where $\Sigma$ is the covariance matrix of $\tilde{r}$; $\phi_M$ is the portfolio weights of the market portfolio; and $\eta_i$ is the $n$-dimensional vector with the $i$th element being 1 and all other elements being 0. Define*

$$1_{n_1,n_2} := (\underbrace{1, \ldots, 1}_{n_1}, \underbrace{0, \ldots, 0}_{n_2})', \tag{12}$$

*then $\tilde{r} - r1_{n_1,n_2}$ is the excess asset return*[6] *and the S-CAPM equations (10) and (11) are equivalent to a single equation*

$$E[\tilde{r}] - r1_{n_1,n_2} = \frac{Cov(\tilde{r}, r_M)}{Var(r_M)}(E[r_M] - r). \tag{13}$$

*Proof.* See E-Companion EC.2.1. □

By incorporating spatial interaction, the S-CAPM generalizes not only the CAPM for ordinary assets but also the CAPM for futures presented in Black (1976) and Duffie (1989, Chapter 4). The S-CAPM can also be extended to the case in which there is no risk-free asset; see E-Companion EC.2.2.

It follows from the S-CAPM equations (10) and (11) that the degree of spatial interaction represented by the parameter $\rho$ affects asset risk premiums in equilibrium because $\Sigma$ is a function of $W$ and $\rho$ (see (8)).

[5] Since the aggregate position of all market participants in a futures contract is zero, $n_1$ needs to be positive in order to ensure that the return of the market portfolio is well defined.

[6] $\tilde{r} - r1_{n_1,n_2}$ is the excess returns of the $n$ assets in the sense that the first $n_1$ elements of $\tilde{r} - r1_{n_1,n_2}$ are the excess returns of the $n_1$ ordinary assets, and the last $n_2$ elements of $\tilde{r} - r1_{n_1,n_2}$ are the returns of the futures contracts, which can be viewed as "excess returns" because futures returns are the payoffs of zero-cost portfolios, just as are the excess returns of ordinary assets.

The S-CAPM implies a zero-intercept constraint on the spatial econometric models for asset returns. Consider the following spatial econometric model, in which the excess returns $\tilde{r} - r1_{n_1,n_2}$ are regressed with a spatial interaction term on the excess return of the market portfolio $r_M - r$:

$$\begin{aligned} \tilde{r} - r1_{n_1,n_2} &= \rho W(\tilde{r} - r1_{n_1,n_2}) + \bar{\alpha} + \beta(r_M - r) + \tilde{\epsilon}, \\ E[\tilde{\epsilon}] &= 0,\ Cov(r_M, \tilde{\epsilon}) = 0. \end{aligned} \tag{14}$$

Then, the S-CAPM implies that, in the above model,

$$\bar{\alpha} = 0. \tag{15}$$

To see this, rewrite (14) as $(I_n - \rho W)(\tilde{r} - r1_{n_1,n_2}) = \bar{\alpha} + \beta(r_M - r) + \tilde{\epsilon}$. Taking covariance with $r_M$ on both sides and using $Cov(r_M, \tilde{\epsilon}) = 0$ yields $\beta = (I_n - \rho W)\frac{Cov(\tilde{r}, r_M)}{Var(r_M)}$, from which it follows that $\bar{\alpha} = (I_n - \rho W)E[(\tilde{r} - r1_{n_1,n_2}) - \frac{Cov(\tilde{r}, r_M)}{Var(r_M)}(r_M - r)]$. If the S-CAPM holds, then (13) implies $\bar{\alpha} = 0$.[7]

# 4. The Spatial Arbitrage Pricing Theory

In this section, we derive the Spatial Arbitrage Pricing Theory (S-APT) and point out its implications. As in Section 2.2, we consider a one-period model with $n$ risky assets. Consider the following factor model with spatial interaction:

$$r_i = \rho \sum_{j=1}^{n} w_{ij} r_j + \alpha_i + \sum_{k=1}^{K} \beta_{ik} f_k + \epsilon_i, i = 1, \ldots, n, \tag{16}$$

where $r_i$, $\rho$, $w_{ij}$, $\alpha_i$, $\epsilon_i$ have the same meaning as in (6); $f_1, \ldots, f_K$ are $K$ risk factors with $E[f_k] = 0$; and $\beta_{ik}$ is the loading coefficient of the asset $i$ on the factor $k$. Let $\tilde{r} := (r_1, \ldots, r_n)'$, $W := (w_{ij})$, $\alpha := (\alpha_1, \ldots, \alpha_n)'$, $B := (\beta_{ik})$, $\tilde{f} := (f_1, \ldots, f_K)'$, and $\tilde{\epsilon} := (\epsilon_1, \ldots, \epsilon_n)'$. Then, the above model can be represented in a vector-matrix form as

$$\tilde{r} = \rho W \tilde{r} + \alpha + B\tilde{f} + \tilde{\epsilon},\ \ E[\tilde{f}] = 0,\ \ E[\tilde{\epsilon}] = 0,\ \ E[\tilde{\epsilon}\tilde{\epsilon}'] = V,\ \ E[\tilde{f}\tilde{\epsilon}'] = 0. \tag{17}$$

The model (17) reduces to the classical APT when $\rho = 0$.

## 4.1. Asymptotic Arbitrage

We first introduce the notion of asymptotic arbitrage defined in Huberman (1982) and in Ingersoll (1984). Suppose the set of factors $\tilde{f} = (f_1, \ldots, f_K)'$ are fixed and consider a sequence of economies with increasing numbers of risky assets whose returns depend on these factors and on spatial interaction. As in Section 3, in the $n$th economy there are $n_1$ ordinary assets and $n_2$ futures

[7] A spatial lag CAPM equation, which is similar to (14) with $\bar{\alpha} = 0$ and considers only ordinary assets but not futures, is *defined* in Fernandez (2011) without theoretical justification. In contrast, the present paper rigorously proves that the S-CAPM relation (13) holds (for both ordinary assets and futures) and that $\bar{\alpha}$ must be 0 in the spatial model (14) under the assumption in Theorem 1.

contracts, where $n = n_1 + n_2$. Suppose the futures prices of the $i$th futures contract are $F_{i,0}^{(n)}$ and $F_{i,1}^{(n)}$ at time 0 and time 1, respectively. As in Section 3, we define the futures returns as

$$r_{n_1+i}^{(n)} = \frac{F_{i,1}^{(n)} - F_{i,0}^{(n)}}{F_{i,0}^{(n)}}, \ i = 1, \cdots, n_2. \tag{18}$$

Assume the returns $\tilde{r}^{(n)} = (r_1^{(n)}, \ldots, r_n^{(n)})'$ are generated by

$$\begin{aligned} \tilde{r}^{(n)} &= \rho^{(n)} W^{(n)} \tilde{r}^{(n)} + \alpha^{(n)} + B^{(n)} \tilde{f} + \tilde{\epsilon}^{(n)}, \text{ where} \\ E[\tilde{f}] = 0, E[\tilde{\epsilon}^{(n)}] &= 0, E[\tilde{\epsilon}^{(n)} (\tilde{\epsilon}^{(n)})'] = V^{(n)}, E[\tilde{f} (\tilde{\epsilon}^{(n)})'] = 0. \end{aligned} \tag{19}$$

The $(n+1)$th economy includes all the $n$ risky assets in the $n$th economy and one extra risky asset. In the $n$th economy, a portfolio is denoted by a vector of dollar-valued positions $h^{(n)} := (h_1^{(n)}, \ldots, h_{n_1}^{(n)}, h_{n_1+1}^{(n)}, \ldots, h_n^{(n)})'$, where $h_1^{(n)}, \ldots, h_{n_1}^{(n)}$ denote the dollar-valued wealth invested in the first $n_1$ assets; $h_{n_1+i}^{(n)} := O_i F_{i,0}^{(n)}$, where $O_i$ denotes the number of $i$th futures contracts held in the portfolio, and $i = 1, \ldots, n_2$. A portfolio $h^{(n)}$ is a zero-cost portfolio if $(h^{(n)})' 1_{n_1,n_2} = 0$, where $1_{n_1,n_2}$ is defined in (12). Then, the payoff of the zero-cost portfolio is $(h^{(n)})'(\tilde{r}^{(n)} + 1_{n_1,n_2}) = (h^{(n)})' \tilde{r}^{(n)}$,[8] because $(h^{(n)})' 1_{n_1,n_2} = 0$.

Asymptotic arbitrage is defined to be the existence of a subsequence of zero-cost portfolios $\{h^{(m_k)}, k = 1, 2, \ldots\}$ and $\delta > 0$ such that

$$E[(h^{(m_k)})' \tilde{r}^{(m_k)}] \geq \delta, \text{ for all } k, \text{ and } \lim_{k \to \infty} Var((h^{(m_k)})' \tilde{r}^{(m_k)}) = 0.^{9} \tag{20}$$

## 4.2. The Spatial Arbitrage Pricing Theory: A Special Case in Which Factors Are Tradable

To obtain a good intuition, we first develop the S-APT in the case in which the factors are the payoff of tradable zero-cost portfolios and there is a risk-free return $r$. Suppose the risk factors $\tilde{f}$ are given by

$$\tilde{f} = \tilde{g} - E[\tilde{g}], \tag{21}$$

where $\tilde{g} = (g_1, g_2, \ldots, g_K)'$ and each $g_k$ is the payoff of a certain tradable zero-cost portfolio. The model (19) can then be written as

$$\tilde{r}^{(n)} - r 1_{n_1,n_2} = \rho^{(n)} W^{(n)} (\tilde{r}^{(n)} - r 1_{n_1,n_2}) + \bar{\alpha}^{(n)} + B^{(n)} \tilde{g} + \tilde{\epsilon}^{(n)}, \tag{22}$$

$$\bar{\alpha}^{(n)} := \alpha^{(n)} - (I_n - \rho^{(n)} W^{(n)}) 1_{n_1,n_2} r - B^{(n)} E[\tilde{g}]. \tag{23}$$

[8] If there is a risk free asset with return $r$, then a zero-cost portfolio with dollar-valued positions $h^{(n)}$ in the risky assets must have a dollar-valued position $-(h^{(n)})' 1_{n_1,n_2}$ in the risk free asset. Then, the payoff of the portfolio is given by $(h^{(n)})'(\tilde{r}^{(n)} - r 1_{n_1,n_2})$.

[9] In the case when there is a risk free asset with return $r$, the term $(h^{(m_k)})' \tilde{r}^{(m_k)}$ should be replaced by $(h^{(m_k)})'(\tilde{r}^{(m_k)} - r 1_{n_1,n_2})$.

THEOREM 2. *Suppose there is a risk-free return $r$ and the risk factors $\tilde{f}$ are given by (21) where $g_1, g_2, \ldots, g_K$ are the payoffs of certain zero-cost portfolios. Suppose*

$$E[\epsilon_i^{(n)}\epsilon_j^{(n)}] = 0, \text{ for } i \neq j; Var(\epsilon_i^{(n)}) \leq \bar{\sigma}^2, \text{ for all } i \text{ and } n, \tag{24}$$

*where $\bar{\sigma}^2$ is a fixed positive number. If there is no asymptotic arbitrage, then*

$$\bar{\alpha}^{(n)} \approx 0, \tag{25}$$

*or, equivalently,*

$$\alpha^{(n)} \approx (I_n - \rho^{(n)} W^{(n)}) 1_{n_1,n_2} r + B^{(n)} E[\tilde{g}]. \tag{26}$$

*The approximation (25) holds in the sense that for any $\delta > 0$ there exists a constant $N_\delta > 0$ such that $N(n,\delta) < N_\delta$ for all $n$, where $N(n,\delta)$ denotes the number of components of $\bar{\alpha}^{(n)}$ whose absolute values are greater than $\delta$.*

*Proof.* See E-Companion EC.3.1. $\square$

The intuition behind the theorem is that if $\tilde{g}$ are the payoffs of zero-cost portfolios, then, by (22), one can construct zero-cost portfolios with payoffs $\bar{\alpha}^{(n)} + \tilde{\epsilon}^{(n)}$ that do not carry systematic risk. If the elements of $\tilde{\epsilon}^{(n)}$ are uncorrelated and have bounded variance, then $\bar{\alpha}^{(n)}$ must be approximately zero; otherwise, one could construct a large zero-cost portfolio with a payoff whose mean would be strictly positive while its variance would vanish, constituting an asymptotic arbitrage opportunity.

### 4.3. The Spatial Arbitrage Pricing Theory: the General Case

THEOREM 3. *(S-APT with Both Ordinary Assets and Futures) Suppose that in the $n$th economy there are $n_1$ ordinary risky assets and $n_2$ futures contracts and the $n_1$ ordinary asset returns and the $n_2$ futures returns are generated by the model (19). If there is no asymptotic arbitrage opportunity, then there is a sequence of factor premiums $\lambda^{(n)} = (\lambda_1^{(n)}, \ldots, \lambda_K^{(n)})'$ and a constant $\lambda_0^{(n)}$, which price all assets approximately:*

$$\alpha^{(n)} \approx (I_n - \rho^{(n)} W^{(n)}) 1_{n_1,n_2} \lambda_0^{(n)} + B^{(n)} \lambda^{(n)}. \tag{27}$$

*The precise meaning of the approximation in (27) is that there exists a positive number $A$ such that the weighted sum of the squared pricing errors is uniformly bounded,*

$$(U^{(n)})'(V^{(n)})^{-1} U^{(n)} \leq A < \infty \text{ for all } n, \tag{28}$$

*where*

$$U^{(n)} = \alpha^{(n)} - (I_n - \rho^{(n)} W^{(n)}) 1_{n_1,n_2} \lambda_0^{(n)} - B^{(n)} \lambda^{(n)}. \tag{29}$$

*In particular, if there exists a risk-free return $r$, then $\lambda_0^{(n)}$ can be identified as $r$.*

*Proof.* See E-Companion EC.3.2. □

Comparing (26) and (27), one can see that the factor risk premiums $\lambda^{(n)}$ in the S-APT can be identified as

$$\lambda^{(n)} = E[\tilde{g}], \tag{30}$$

if $\tilde{f} = \tilde{g} - E[\tilde{g}]$ and $\tilde{g}$ are the payoffs of zero-cost traded portfolios. The S-APT implies that the degree of spatial interaction affects asset risk premiums. Indeed, let $(\bar{\beta}^{(n)}_{i,1}, \bar{\beta}^{(n)}_{i,2}, \ldots, \bar{\beta}^{(n)}_{i,K})$ be the $i$th row of $(I_n - \rho^{(n)} W^{(n)})^{-1} B^{(n)}$. Then, (27) implies that the ordinary assets are approximately priced by

$$E[r^{(n)}_i] - \lambda^{(n)}_0 \approx \sum_{k=1}^{K} \bar{\beta}^{(n)}_{i,k} \lambda^{(n)}_k, i = 1, \ldots, n_1 \tag{31}$$

and that the futures contracts are approximately priced by

$$\frac{E[F^{(n)}_{i,1}] - F^{(n)}_{i,0}}{F^{(n)}_{i,0}} \approx \sum_{k=1}^{K} \bar{\beta}^{(n)}_{n_1+i,k} \lambda^{(n)}_k,\ i = 1, \ldots, n_2. \tag{32}$$

(31) and (32) show that the expected returns of both ordinary assets and futures contracts are affected by the spatial interaction parameter $\rho$ because, for all $j$ and $k$, $\bar{\beta}^{(n)}_{j,k}$ depends on the spatial interaction terms $\rho^{(n)}$ and $W^{(n)}$.

### 4.4. Comparison with the SAR Model

The spatial autoregressive (SAR) model (see, e.g., Lesage and Pace (2009, Chapter 2.6)) is one of the most commonly adopted models in the spatial econometrics literature. The SAR model postulates that the dependent variables (usually prices or log prices of assets) $y_1, \ldots, y_n$ are generated by

$$y_i = \rho \sum_{j=1}^{n} w_{ij} y_j + \beta_0 + \sum_{k=1}^{K} \beta_k x_{ik} + \epsilon_i, i = 1, \ldots, n, \tag{33}$$

where $\rho$, $w_{ij}$, and $\epsilon_i$ have the same meaning as in (5); $x_{ik}$ are explanatory variables; $\beta_0$ is the intercept; and $\beta_1, \ldots, \beta_K$ are coefficients in front of explanatory variables.

Although the first term in (33) of the SAR model is the same as the first term of the S-APT model (16), there are substantial differences between the two: (i) In terms of model specification, the S-APT imposes a linear constraint on model parameters ((26) or (27)), while the parameters in the SAR model are free parameters. (ii) The S-APT model is a common factor model but the SAR model is an individual factor model. In the SAR model (33), the factors $x_{ik}$ are individual factors that may be different for different $i$, but the intercept $\beta_0$ and factor loading $\beta_k$ are the same for different $i$. In contrast, in the S-APT model (16), the factors $f_k$ are common factors that are the same for different $i$, but the intercepts $\alpha_i$ and the factor loadings $\beta_{ik}$ are different for different $i$.

## 5. Statistical Inference for S-APT

Let $\tilde{r}_t = (r_{1t}, r_{2t}, \ldots, r_{nt})'$ be the observation of $n = n_1 + n_2$ asset returns that consist of $n_1$ ordinary asset returns and $n_2$ futures returns in the $t$th period. Let $r_{ft}$ be the risk free return in the $t$th period. Let $\tilde{y}_t = (y_{1t}, y_{2t}, \ldots, y_{nt})' := \tilde{r}_t - r_{ft} 1_{n_1,n_2}$ denote the excess asset returns. Let $\tilde{g}_t = (g_{1t}, g_{2t}, \ldots, g_{Kt})'$ be the observation of the $k$ factors in the $t$th period (note that $E[\tilde{g}_t]$ may not be zero).

Assume $\tilde{y}_t$ and $\tilde{g}_t$ are generated by the following panel data model, a multi-period version of the model (22):

$$\begin{aligned}
&\tilde{y}_t = \rho W \tilde{y}_t + \bar{\alpha} + B\tilde{g}_t + \tilde{\epsilon}_t, t = 1, 2, \ldots, T, \\
&(\tilde{y}_t, \tilde{g}_t), t = 1, 2, \ldots, T, \text{ are i.i.d.,} \qquad (34) \\
&\tilde{\epsilon}_t \mid \tilde{g}_t \sim N(0, \sigma^2 I_n).
\end{aligned}$$

The model (34) incorporates three features: (i) a spatial lag in the dependent variables, (ii) individual-specific fixed effects, and (iii) heterogeneity of factor loadings on common factors. However, existing models have as yet incorporated only some but not all these features. Lee and Yu (2010a) investigate the asymptotic properties of the QMLEs for spatial panel data models that incorporate the features (i) and (ii) but not (iii); Holly, Pesaran, and Yamagata (2010) and Pesaran and Tosetti (2011) consider panel data models that incorporate spatially correlated cross-section errors and the features (ii) and (iii), but not (i); see Anselin, Le Gallo, and Jayet (2008) and Lee and Yu (2010b) for more comprehensive discussion of spatial panel data models and the asymptotic properties of MLE and QMLE for these models.

For the brevity of notation, we define

$$\underline{b} := (\bar{\alpha}_1, \beta_{11}, \beta_{12}, \ldots, \beta_{1K}, \ldots, \bar{\alpha}_n, \beta_{n1}, \beta_{n2}, \ldots, \beta_{nK})', \qquad (35)$$

where $\beta_{ik}$ is the $(i,k)$ element of $B$. Denote the parameter vector of the model as $\theta := (\rho, \underline{b}', \sigma^2)'$. Let $\theta_0 = (\rho_0, \underline{b}_0', \sigma_0^2)'$ be the true model parameters.

Following Manski (1995), we study both the identification and the statistical inference problems for the proposed spatial econometric model. Manski (1995, p. 4) points out that "it is useful to separate the inferential problem into statistical and identification components. Study of identification seek to characterize the conclusions that could be drawn if one could use the sampling process to obtain an unlimited number of observations." Furthermore, Manski (1995, p. 6) emphasizes that "the study of identification logically comes first" because "negative identification findings imply that statistical inference is fruitless: it makes no sense to try to use a sample of finite size to infer something that could not be learned even if a sample of infinite size were available." In our S-APT model in (34), it is not apparent whether all components of the parameters $\theta$ are identifiable given the observation $(\tilde{y}_t, \tilde{g}_t)$, $t = 1, \ldots, T$; hence, we will first clarify the identifiability issue in Section 5.1 below.

### 5.1. Identifiability of Model Parameters

The model parameters $\theta_0$ are identifiable if the spatial weight matrix $W$ is regular (i.e., satisfying simple regularity conditions that are easy to check); see E-Companion EC.4.1 for detailed discussion. It can be easily checked that in all empirical examples of this paper, $W$ is regular. In the rest of the section, we assume that $W$ is regular and hence $\theta_0$ is identifiable.[10]

### 5.2. Model Parameter Estimation

The model parameters can be estimated by conditional maximum likelihood estimates (MLE). Let

$$X_t := \begin{pmatrix} 1, g_{1t}, \cdots, g_{Kt} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & 1, g_{1t}, \cdots, g_{Kt} \end{pmatrix} \in \mathbb{R}^{n \times n(K+1)}. \tag{36}$$

Then, the conditional log likelihood function of the model is given by

$$\ell(\theta) = \ell(\rho, \underline{b}, \sigma^2) := \sum_{t=1}^{T} l(\tilde{y}_t \mid \tilde{g}_t, \theta), \text{ where} \tag{37}$$

$$\begin{aligned} l(\tilde{y}_t \mid \tilde{g}_t, \theta) = &-\frac{n}{2}\log(2\pi\sigma^2) + \frac{1}{2}\log(\det((I_n - \rho W')(I_n - \rho W))) \\ &- \frac{1}{2\sigma^2}(\tilde{y}_t - \rho W \tilde{y}_t - X_t \underline{b})'(\tilde{y}_t - \rho W \tilde{y}_t - X_t \underline{b}). \end{aligned} \tag{38}$$

The likelihood defined in (37) is a conditional likelihood. In the S-APT model specified in (34), we observe $(\tilde{y}_t, \tilde{g}_t)$, $t = 1, \ldots, T$ and we would like to estimate the parameter $\theta$, which only affects the conditional distribution of $\tilde{y}_t$ given $\tilde{g}_t$. The S-APT model specifies the conditional distribution of $\tilde{y}_t$ given $\tilde{g}_t$ but not the marginal distribution of $\tilde{g}_t$, which is not of interest in our problem. Hence, we adopt the conditional maximum likelihood method.

Let $[\zeta, \gamma]$ be an interval such that $\zeta < 0 < \gamma$ and $I_n - \rho W$ is invertible for $\rho \in [\zeta, \gamma]$.[11] It can be shown[12] that the conditional MLE $\hat{\theta} = (\hat{\rho}, \hat{\underline{b}}', \widehat{\sigma^2})'$ is given by

$$\hat{\rho} = \arg\max_{\rho \in [\zeta, \gamma]} \ell_c(\rho), \hat{\underline{b}} = b(\hat{\rho}), \widehat{\sigma^2} = s(\hat{\rho}),$$

[10] In fact, if $W$ is not regular, then the elements of $W$ satisfy $n(n+1)/2$ constraints given by (EC.32) and (EC.33) in E-Companion EC.4.1; hence, unless $W$ is carefully constructed to satisfy these constraints, $W$ is regular and the (unknown) true parameter is identifiable. For example, when $W$ is not regular and $n = 3$, $W$ has six off-diagonal elements that satisfy six constraints; hence, only very special $W$ are not regular.

[11] For any $W$, because $\lim_{\rho \to 0} \det(I_n - \rho W) = 1$, there always exists an interval $[\zeta, \gamma]$ such that $\zeta < 0 < \gamma$ and that $I_n - \rho W$ is invertible for $\rho \in [\zeta, \gamma]$. In fact, $I_n - \rho W$ is invertible if and only if $\rho^{-1}$ is not an eigenvalue of $W$ (see footnote 4). Hence, the specification of $[\zeta, \gamma]$ depends on $W$: (i) If $W$ has at least two different real eigenvalues and $\omega_{\min} < 0 < \omega_{\max}$ are the minimum and maximum real eigenvalues, then $[\zeta, \gamma]$ can be chosen as an interval that lies inside $(\omega_{\min}^{-1}, \omega_{\max}^{-1})$. In particular, if the rows of $W$ are normalized to sum up to 1, which is commonly seen in spatial econometrics literature, then $\omega_{\max} = 1$. (ii) If $W$ does not have real eigenvalues, then $[\zeta, \gamma]$ can be any interval contains 0. See Lesage and Pace (2009, Chapter 4.3.2, p.88) for more detailed discussion.

[12] Note that for any given $\rho$, the original model can be rewritten as $\tilde{y}_t - \rho W \tilde{y}_t = X_t \underline{b} + \tilde{\epsilon}_t$, $t = 1, 2, \ldots, T$, from which the classical theory of linear regression shows that $\underline{b} = b(\rho)$ and $\sigma^2 = s(\rho)$ maximize the conditional log likelihood function (37). Because $\ell_c(\rho) = \ell(\rho, b(\rho), s(\rho))$ and $\hat{\rho}$ maximizes $\ell_c(\rho)$, it follows that $\hat{\rho}$, $\hat{\underline{b}} = b(\hat{\rho})$, and $\widehat{\sigma^2} = s(\hat{\rho})$ maximize $\ell(\rho, \underline{b}, \sigma^2)$, i.e., they are the conditional MLE.

where

$$\begin{aligned}\ell_c(\rho) &:= \ell(\rho, b(\rho), s(\rho)) \\ &= -\frac{nT}{2}\log(2\pi s(\rho)) + \frac{T}{2}\log(\det((I_n - \rho W')(I_n - \rho W))) - \frac{nT}{2}, \\ b(\rho) &:= \left(\sum_{t=1}^{T} X_t' X_t\right)^{-1} \sum_{t=1}^{T} X_t'(I_n - \rho W)\tilde{y}_t, \\ s(\rho) &:= \frac{1}{nT}\sum_{t=1}^{T}((I_n - \rho W)\tilde{y}_t - X_t b(\rho))'((I_n - \rho W)\tilde{y}_t - X_t b(\rho)).\end{aligned} \tag{39}$$

Although we use the conditional MLE, Theorem EC.2 in E-Companion EC.4.2 shows that the conditional MLE estimates have consistency and asymptotic normality; in addition, the simulation studies in E-Companion EC.4.2 show that the conditional MLE achieves accurate estimation results.

### 5.3. Hypothesis Test and Goodness of Fit of the Model (34)

For simplicity, we assume that the factors $\tilde{g}$ are the payoffs of zero-cost tradable portfolios. In this case, Theorem 2 shows that the S-APT imposes an approximate zero-intercept constraint $\bar{\alpha}^{(n)} \approx 0$ (see Eq. (25)). As in the classical factor pricing literature, we test the S-APT by testing the exact zero-intercept constraint

$$H_0 : \bar{\alpha}_0 = 0; \;\; H_1 : \bar{\alpha}_0 \neq 0, \tag{40}$$

where $\bar{\alpha}_0$ is the true parameter in the model (34).

We can test the hypothesis using conditional likelihood ratio test statistics. Under the null hypothesis, the conditional likelihood ratio test statistic

$$LR = 2\left[\sum_{t=1}^{T} l(\tilde{y}_t \mid \tilde{g}_t, \hat{\theta}) - \sum_{t=1}^{T} l(\tilde{y}_t \mid \tilde{g}_t, \theta^*)\right] \tag{41}$$

has an asymptotic $\chi^2(n)$ distribution.[13] Here, $\sum_{t=1}^{T} l(\tilde{y}_t \mid \tilde{g}_t, \hat{\theta})$ denotes the conditional log likelihood function evaluated at $\hat{\theta}$, which is the conditional MLE of parameters estimated with no constraints; while $\sum_{t=1}^{T} l(\tilde{y}_t \mid \tilde{g}_t, \theta^*)$ its counterpart evaluated at the conditional MLE $\theta^*$ estimated under the constraint that the null holds (i.e., $\bar{\alpha}_0 = 0$). The conditional likelihood ratio test is asymptotically equivalent to the traditional tests in asset pricing, such as the Gibbons-Ross-Shanken test; see Gibbons, Ross, and Shanken (1989) as well as Chapter 1 and 2 in Hayashi (2000).

[13] This can be shown by verifying the conditions of Proposition 7.11 in Hayashi (2000, p. 494). Let $a(\theta) := \bar{\alpha}$. Then, the Jacobian $\frac{\partial a(\theta_0)}{\partial \theta'}$ is of full row rank. We then need to verify the conditions of Proposition 7.9 in Hayashi (2000, p. 475), but it is done in the proof of Theorem EC.2 in E-Companion EC.4.2 of this paper.

The goodness of fit of the model can be evaluated by adjusted $R^2$. The theoretical adjusted $R^2$ of the $i$th asset in the model (34) is defined as

$$R_i^2 = 1 - \frac{T-1}{T-K-1}\frac{Var(\epsilon_i)}{Var(y_i)},\ i = 1, 2, \ldots, n, \tag{42}$$

where $Var(\epsilon_i) = \sigma_0^2$ and $Var(y_i)$ is equal to the $i$th diagonal element of the covariance matrix $(I_n - \rho_0 W)^{-1} B_0 \cdot Cov(\tilde{g}) \cdot B_0'(I_n - \rho_0 W')^{-1} + \sigma_0^2 (I_n - \rho_0 W)^{-1}(I_n - \rho_0 W')^{-1}$. The sample adjusted $R^2$ of the $i$th asset is calculated using (42) with $Var(\epsilon_i)$ and $Var(y_i)$ replaced by their respective sample counterparts.

For a simulation study of the likelihood ratio test and the adjusted $R^2$, see E-Companion EC.4.3.

## 6. Application 1: Eurozone Stock Indices

In this section, we continue to study factor models for European stock indices returns[14] considered in the motivating example in Section 2.1.

### 6.1. The Data

The data of the European stock indices returns are the same as those used in Section 2.1. We take four factors used in Fama and French (2012), namely the market factor (MKT), the size factor (SMB), the value factor (HML) and the momentum factor (MOM). The factors aforementioned have long been documented in the literature to account for the majority of comovements of equities returns. By including these four factors in the S-APT model, the empirical study is designed to more clearly reveal the contribution of spatial interaction in relation to what has been understood in the literature. Since the returns in Fama and French (2012) are sorted by corporate characteristics, while here stocks are implicitly sorted by locations/nations, one may expect some additional factors. We construct a new risk factor that is related to sovereign credit risk. Table EC.3 in E-Companion EC.5 shows the S&P credit ratings of the 11 countries during 2001–2013, where it can be seen that Germany is the only country that has maintained top-notch AAA rating, while Greece, Ireland, Italy, Portugal, and Spain are the only countries which were ever rated BBB+ or below during that

[14] To test APT, Fama and French (2012) constructed 25 size-B/M portfolios consisting of individual stocks in the region of Europe. The monthly returns of the 25 size-B/M test portfolios, used in the left-hand-side of the regression, are formed by the intersection of a sort on the size (market capitalization) of individual stocks in the European region and a sort on the BE/ME ratio of those stocks. As each of the 25 test portfolios may include firms from different countries in the European region, it may be difficult to characterize the "location" of each test portfolio, and hence it is not clear how to define the spatial weight matrix that describe the spatial interaction between the 25 test portfolios. Therefore, the S-APT model may not be directly applicable to the returns of the 25 test portfolios. However, for each test portfolio, one may further divide it into subportfolios that only include firms in a particular country; then, the location of such subportfolios can be clearly identified. We can then use the returns of these subportfolios as the left-hand asset returns in the S-APT model for testing the model, although the sample sizes in the subportfolios are much smaller. It would be interesting to see how the S-APT model perform for these subportfolios; we will leave this for future research.

period. Thus, we introduce the credit factor as the difference between the return of stock index of Germany and the average returns of indices of Greece, Ireland, Italy, Portugal, and Spain:

$$g_{credit} := r_{Germany} - \frac{1}{5}(r_{Greece} + r_{Ireland} + r_{Italy} + r_{Portugal} + r_{Spain}). \tag{43}$$

Similar to the value factor constructed in Fama and French (1993), which is a proxy for the distress factor in the corporate domain, the risk factor (43) may be considered as a proxy of the sovereign version of distress risk.

### 6.2. Empirical Performance of S-APT and APT Models

We investigate the performance of the following six APT models and their S-APT counterparts:

- Model 1: APT with MKV, SMB, HML, and MoM factor (Fama and French (2012))
- Model 2: APT with MKV, SMB, HML, MoM, and Credit factor
- Model 3: APT with MKV, MoM, and Credit factor
- Model 4: APT with MKV, SMB, MoM, and Credit factor
- Model 5: APT with MKV, HML, MoM, and Credit factor
- Model 6: APT with MKV, SMB, HML, and Credit factor
- Model 1s: S-APT with MKV, SMB, HML, and MoM factor
- Model 2s: S-APT with MKV, SMB, HML, MoM, and Credit factor
- Model 3s: S-APT with MKV, MoM, and Credit factor
- Model 4s: S-APT with MKV, SMB, MoM, and Credit factor
- Model 5s: S-APT with MKV, HML, MoM, and Credit factor
- Model 6s: S-APT with MKV, SMB, HML, and Credit factor

An "APT" model means the type of factor model considered in Fama and French (2012) in which heterogeneous variances of residuals for different returns are assumed. The model 1 is the same as the model specified in the motivating example in Section 2.1. The S-APT model is specified in (34) with homogeneous variances for residuals. In the S-APT models, the spatial weight matrix $W$ is defined as $W_{ij} := (s_i d_{ij})^{-1}$ for $i \neq j$ and $W_{ii} = 0$, where $d_{ij}$ is the driving distance between the capital of country $i$ and that of country $j$ and $s_i := \sum_j d_{ij}^{-1}$.

The no asymptotic arbitrage (i.e., zero-intercept) hypothesis test for the APT models is specified in (3), and that for the S-APT models is specified in (40). Table 3 shows the p-value for testing the zero-intercept hypothesis, the number of parameters, the Akaike information criterion (AIC), the 95% confidence interval (C.I.) of $\rho_0$ (only for the S-APT models), and the 95% C.I. of $\kappa$ defined in Eq. (2) for the residuals for all the above models. The domain of $\rho_0$ and $\kappa$ in the conditional MLE estimation is $[-2.5342, 1]$. The only models that are not rejected in the test of zero-intercept hypothesis are model 3s and 4s, which incorporate spatial interaction. The models 3s and 4s also

| Model | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| p-value | 0.0 | 0.0 | 0.0002 | 0.0005 | 0.0 | 0.0 |
| AIC | 10150 | 9841 | 10048 | 9955 | 9939 | 9889 |
| number of parameters | 66 | 77 | 55 | 66 | 66 | 66 |
| 95% C.I. of $\kappa$ for residuals | [0.02, 0.21] | [-0.05, 0.14] | [-0.01, 0.18] | [0.01, 0.20] | [-0.08, 0.12] | [-0.04, 0.15] |
| **Model** | **1s** | **2s** | **3s** | **4s** | **5s** | **6s** |
| p-value | 0.0002 | 0.0099 | **0.1854** | **0.2912** | 0.0078 | 0.0002 |
| AIC | 8826 | 8625 | **8793** | **8730** | 8696 | 8668 |
| number of parameters | 57 | 68 | **46** | **57** | 57 | 57 |
| 95% C.I. of $\rho_0$ | [0.06, 0.25] | [ -0.01, 0.18] | [**0.01**, **0.19**] | [**0.03**, **0.21**] | [-0.04, 0.16] | [0.01, 0.19] |
| 95% C.I. of $\kappa$ for residuals | [-0.08, 0.12] | [-0.09, 0.11] | [**-0.09**, **0.11**] | [**-0.08**, **0.11**] | [-0.10, 0.10] | [-0.09, 0.11] |

**Table 3 The p-value for testing the no asymptotic arbitrage (i.e., zero-intercept) hypothesis, the Akaike information criterion (AIC), the number of parameters, the 95% confidence interval (C.I.) of $\rho_0$ (only for the S-APT models), and the 95% C.I. of $\kappa$ defined in (2) for the residuals of different models for the eurozone stock indices returns. Model 3s and Model 4s appear to perform better than the other models.**

have better performance than the APT models in terms of AIC. In general, the S-APT models seem to eliminate the spatial correlation among regression residuals more effectively than the APT models, as indicated by $\kappa$. Furthermore, for all the S-APT models that are not rejected (Models 3s and 4s), $\rho_0$ are found to be significantly positive. Note that adding the spatial term seems to improve the p-value and the AIC of its counterpart model without the spatial term. While the Credit, MoM, and SMB factors all seem to play a role in explaining return comovements, adding the HML factor in the model does not seem to improve model performance in terms of p-value and AIC. The adjusted $R^2$ of fitting Model 4s with the zero-intercept constraint is reported in Figure 2. The estimates of the parameters of the S-APT model 4s are reported in E-Companion EC.5.

Besides the AIC and adjusted $R^2$ above, we also perform diagnostics of residuals of the S-APT model 4s. (1) Table 4 shows the correlation matrix of the stock indices returns $\tilde{y}_t$ in (34) and that of the residuals $\tilde{\epsilon}_t$ in (34) for the S-APT model 4s. It seems that the S-APT model 4s is effective in *eliminating cross-sectional correlations*. (2) Table 5 shows the sample autocorrelation functions (ACFs) of $\tilde{y}_t$ and that of $\tilde{\epsilon}_t$ of the S-APT model 4s. It seems that the S-APT model 4s is effective in *reducing the autocorrelations* in $\tilde{y}_t$. (3) Table 6 shows the p-values of Kolmogorov-Smirnov test for

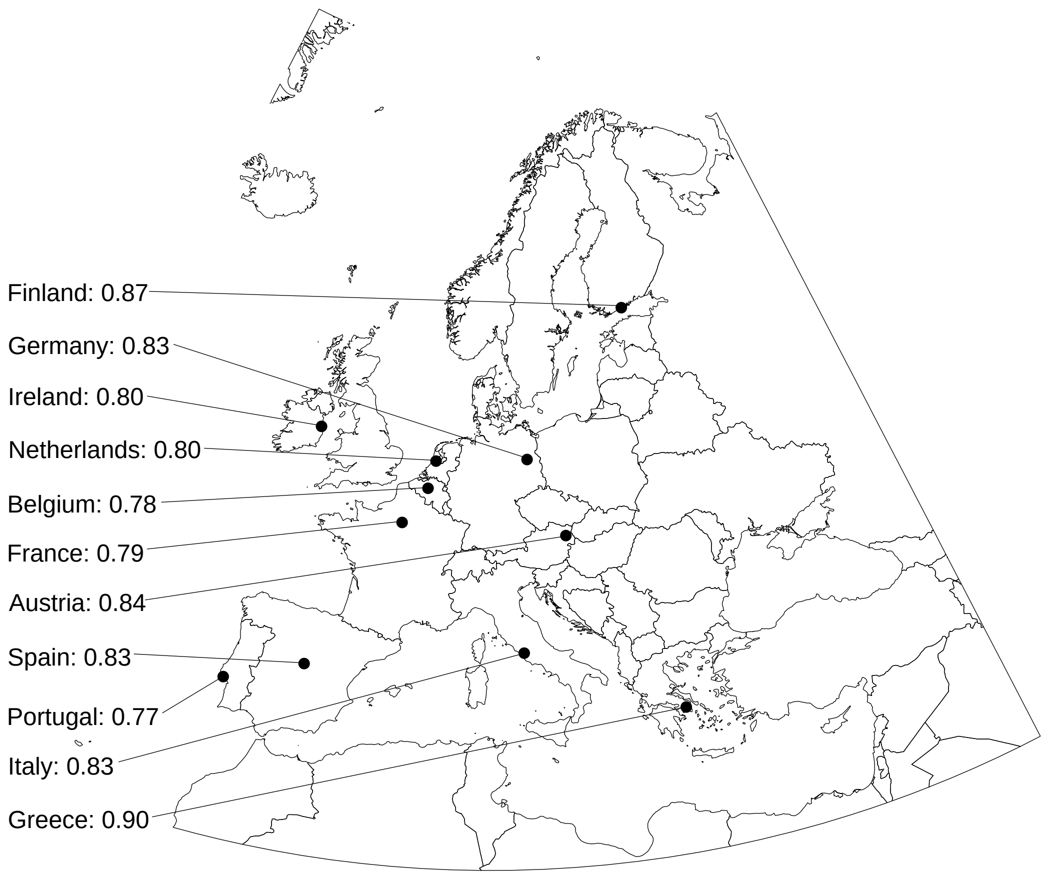



**Figure 2** **The adjusted $R^2$ of fitting the Model 4s with zero-intercept constraint to the eurozone stock indices.**

normal distribution of the residuals $\tilde{\epsilon}_t$ of the S-APT model 4s. The hypothesis that the residuals have normal distributions is rejected at 5% level only for 1 out of 11 stock indices returns .

### 6.3. Robustness Check

The empirical results reported above seem to be robust with respect to different specifications of spatial matrix $W$. In Table 7 we compare the estimation and testing results using two definitions of $W$ in the S-APT model 4s: (i) $W_{ij} := (s_i d_{ij})^{-1}$ where $d_{ij}$ is the geographic distance[15] between the capital of the $i$th country and that of the $j$th country and $s_i := \sum_{j \neq i} d_{ij}^{-1}$. In this case, the domain of $\rho_0$ in conditional MLE estimation is $[-2.6399, 1]$. (ii) $W_{ij} := (s_i d_{ij})^{-1}$ and $d_{ij}$ is the driving distance. The numerical values of $W$ can be found in E-Companion EC.5.

The empirical results reported above also seem to be robust with respect to the specification of the credit factor. We consider the following four alternative definitions of the credit factor.

[15] The geographic distance is calculated from the longitude and latitude coordinates using the Vincenty's formulae (Vincenty (1975)), which assumes that the figure of the earth is an oblate spheroid instead of a sphere.

| | Austria | Belgium | Finland | France | Germany | Greece | Ireland | Italy | Netherlands | Portugal | Spain |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Austria | 1.0000 | | | | | | | | | | |
| Belgium | 0.8639$^{**}$ | 1.0000 | | | | | | | | | |
| Finland | 0.6165$^{**}$ | 0.6834$^{**}$ | 1.0000 | | | | | | | | |
| France | 0.8004$^{**}$ | 0.8998$^{**}$ | 0.8151$^{**}$ | 1.0000 | | | | | | | |
| Germany | 0.7388$^{**}$ | 0.8417$^{**}$ | 0.7575$^{**}$ | 0.9448$^{**}$ | 1.0000 | | | | | | |
| Greece | 0.7827$^{**}$ | 0.7760$^{**}$ | 0.6605$^{**}$ | 0.7891$^{**}$ | 0.7528$^{**}$ | 1.0000 | | | | | |
| Ireland | 0.7808$^{**}$ | 0.8216$^{**}$ | 0.6736$^{**}$ | 0.8111$^{**}$ | 0.7785$^{**}$ | 0.6726$^{**}$ | 1.0000 | | | | |
| Italy | 0.7947$^{**}$ | 0.8662$^{**}$ | 0.7682$^{**}$ | 0.9403$^{**}$ | 0.8788$^{**}$ | 0.8352$^{**}$ | 0.7854$^{**}$ | 1.0000 | | | |
| Netherlands | 0.8001$^{**}$ | 0.8920$^{**}$ | 0.7581$^{**}$ | 0.9389$^{**}$ | 0.9120$^{**}$ | 0.7556$^{**}$ | 0.8176$^{**}$ | 0.8806$^{**}$ | 1.0000 | | |
| Portugal | 0.7848$^{**}$ | 0.8444$^{**}$ | 0.6883$^{**}$ | 0.8483$^{**}$ | 0.7725$^{**}$ | 0.7705$^{**}$ | 0.7292$^{**}$ | 0.8513$^{**}$ | 0.7992$^{**}$ | 1.0000 | |
| Spain | 0.7561$^{**}$ | 0.8309$^{**}$ | 0.6980$^{**}$ | 0.8984$^{**}$ | 0.8325$^{**}$ | 0.8182$^{**}$ | 0.7306$^{**}$ | 0.9128$^{**}$ | 0.8382$^{**}$ | 0.8416$^{**}$ | 1.0000 |

| | Austria | Belgium | Finland | France | Germany | Greece | Ireland | Italy | Netherlands | Portugal | Spain |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Austria | 1.0000 | | | | | | | | | | |
| Belgium | 0.2981$^{**}$ | 1.0000 | | | | | | | | | |
| Finland | $-0.2298^{**}$ | $-0.2963^{**}$ | 1.0000 | | | | | | | | |
| France | -0.0733 | 0.0571 | 0.1132 | 1.0000 | | | | | | | |
| Germany | -0.0504 | 0.0185 | $-0.2069^{*}$ | 0.2467$^{**}$ | 1.0000 | | | | | | |
| Greece | 0.1406 | -0.0335 | -0.0423 | 0.0377 | 0.6520$^{**}$ | 1.0000 | | | | | |
| Ireland | 0.0650 | 0.0817 | -0.0699 | -0.1337 | 0.0305 | $-0.2962^{**}$ | 1.0000 | | | | |
| Italy | -0.0563 | -0.0574 | -0.0477 | 0.3498$^{**}$ | 0.4636$^{**}$ | 0.1401 | -0.1723 | 1.0000 | | | |
| Netherlands | 0.0106 | 0.1220 | -0.1002 | 0.0566 | -0.0702 | -0.0283 | 0.0677 | -0.0532 | 1.0000 | | |
| Portugal | -0.0119 | 0.1384 | -0.0673 | 0.1704 | 0.2954$^{**}$ | -0.0871 | $-0.2120^{*}$ | 0.0201 | -0.0723 | 1.0000 | |
| Spain | -0.0956 | -0.0610 | $-0.2250^{**}$ | 0.2179$^{*}$ | 0.5114$^{**}$ | 0.1489 | $-0.2553^{**}$ | 0.2237$^{**}$ | -0.0480 | 0.0546 | 1.0000 |

**Table 4 The top matrix is the sample correlation matrix of the eurozone stock indices returns $\tilde{y}_t$ in (34); the bottom one is that of the residuals $\tilde{\epsilon}_t$ in (34) for the S-APT model 4s. The 95% significance is marked by $*$ while 99% $**$. It seems that the S-APT model 4s is effective in eliminating cross-sectional correlations.**

- Definition 1: $g_{credit} := r_{Germany} - \frac{1}{5}(r_{Greece} + r_{Ireland} + r_{Italy} + r_{Portugal} + r_{Spain})$ (the definition used in Section 6.2).
- Definition 2: $g_{credit} := \frac{1}{2}(r_{Germany} + r_{Finland}) - \frac{1}{5}(r_{Greece} + r_{Ireland} + r_{Italy} + r_{Portugal} + r_{Spain})$.
- Definition 3: $g_{credit} := r_{Germany} - r_{Greece}$ .
- Definition 4: $g_{credit} := r_{Germany} - \frac{1}{2}(r_{Greece} + r_{Spain})$.

Table 8 shows that the estimation and testing results for fitting the S-APT model 4s to the stock indices returns under the four definitions of credit factor are similar.

# 7. Application 2: S&P/Case-Shiller Home Price Indices Futures

## 7.1. Data

The S&P/Case-Shiller Home Price Indices (CSI Indices) are constructed based on the method proposed by Case and Shiller (1987) and are the leading measure of single family home prices in the United States. The CSI index family includes twenty indices for twenty metropolitan statistical areas (MSAs) and three composite indices (National, 10-City, and 20-City). The indices are updated monthly, except for the national index, which is updated quarterly. The CSI Indices themselves are not directly traded; however, CSI Indices futures are traded at the Chicago Mercantile Exchange. There are, in total, eleven CSI Indices futures contracts; one is written on the composite 10-City CSI Index and the other ten are on the CSI Indices of ten MSAs: Boston, Chicago, Denver, Las Vegas, Los Angeles, Miami, New York, San Diego, San Francisco, and Washington, D.C. On any

| | Lag-1 | Lag-2 | Lag-3 | Lag-4 | Lag-5 |
|---|---|---|---|---|---|
| Austria | $0.2490^{**}$ | 0.1165 | 0.1129 | 0.0773 | -0.0968 |
| Belgium | $0.2271^{**}$ | -0.0111 | 0.0397 | 0.1806 | 0.0541 |
| Finland | $0.2050^{*}$ | $-0.1635^{*}$ | 0.0170 | 0.0129 | 0.0096 |
| France | 0.0893 | -0.0691 | 0.0818 | 0.1158 | -0.0303 |
| Germany | 0.0878 | -0.0922 | 0.0848 | 0.0477 | 0.0060 |
| Greece | 0.1551 | -0.0015 | $0.1864^{*}$ | 0.0202 | -0.0067 |
| Ireland | $0.2414^{**}$ | 0.0962 | 0.1568 | $0.1741^{*}$ | 0.0115 |
| Italy | 0.0847 | -0.1293 | $0.1603^{*}$ | 0.1413 | -0.0847 |
| Netherlands | 0.1077 | -0.0053 | 0.0844 | 0.0705 | -0.0151 |
| Portugal | $0.2097^{**}$ | -0.0324 | 0.1170 | $0.1706^{*}$ | 0.0116 |
| Spain | 0.0588 | -0.0583 | 0.1280 | 0.0342 | -0.0646 |

| | Lag-1 | Lag-2 | Lag-3 | Lag-4 | Lag-5 |
|---|---|---|---|---|---|
| Austria | -0.0325 | 0.0141 | 0.0435 | -0.0062 | 0.0629 |
| Belgium | 0.0584 | $-0.3186^{**}$ | -0.1727 | 0.1005 | 0.0875 |
| Finland | $0.1814^{*}$ | $-0.2197^{**}$ | $-0.1779^{*}$ | -0.1414 | 0.0139 |
| France | -0.1191 | 0.0783 | -0.0218 | -0.0211 | -0.0588 |
| Germany | 0.0495 | -0.1382 | 0.1311 | -0.0329 | 0.0405 |
| Greece | -0.0624 | -0.0736 | 0.1461 | -0.0038 | 0.0132 |
| Ireland | 0.0146 | 0.0982 | 0.0760 | 0.0168 | 0.1248 |
| Italy | -0.1313 | -0.1108 | 0.0992 | -0.0720 | 0.0862 |
| Netherlands | -0.0575 | -0.0830 | -0.0282 | $-0.3092^{**}$ | -0.0409 |
| Portugal | 0.0638 | -0.0833 | -0.0148 | -0.0723 | 0.0620 |
| Spain | -0.0453 | -0.0221 | 0.1016 | -0.0371 | -0.0195 |

**Table 5 The top matrix shows the sample autocorrelation functions (ACFs) for lags up to 5 of the eurozone stock indices returns $\tilde{y}_t$ in (34); the bottom one shows those of the residuals $\tilde{\epsilon}_t$ in (34) for the S-APT model 4s. The $95\%$ significance is marked by $*$ while $99\%$ is marked by $**$. It seems that the S-APT model 4s is effective in reducing the autocorrelations in $\tilde{y}_t$.**

| Country | Austria | Belgium | Finland | France | Germany | Greece | Ireland | Italy | Netherlands | Portugal | Spain |
|---|---|---|---|---|---|---|---|---|---|---|---|
| p-value | 0.5339 | 0.4355 | $0.0338^{*}$ | 0.5775 | 0.8218 | 0.6697 | 0.9327 | 0.5062 | 0.9805 | 0.8728 | 0.8680 |

**Table 6 P-values of Kolmogorov-Smirnov test for the normal distribution of the residuals $\tilde{\epsilon}_t$ in (34) for the S-APT model 4s for the eurozone stock indices returns. Low p-values (lower than 5%) are marked with asterisks. The hypothesis that the residuals have normal distributions is rejected only for 1 out of 11 indices returns.**

given day, the futures contract with the nearest maturity among all the traded futures contracts is called the first nearest-to-maturity contract. In the empirical study, we use the first nearest-to-maturity futures contract to define one-month return of futures because this contract usually has better liquidity than the others. The time period of the data is from June 2006 to February 2014.

We consider three factors for the CSI indices futures. First, we construct a factor related to credit risk, as the credit risk may be a proxy of the risk of public finance (e.g. state pension schemes and infrastructure improvements). Table EC.4 in E-Companion EC.5 shows the S&P credit ratings of

| $W$ | p-value for testing $\bar{\alpha}_0 = 0$ | C.I. of $\rho_0$ | adjusted $R^2$ | | | | |
|---|---|---|---|---|---|---|---|
| | | | Austria | Belgium | Finland | France | Germany |
| Geographic distance | 0.2866 | [0.022, 0.204] | 0.8404 | 0.7756 | 0.8676 | 0.7858 | 0.8304 |
| Driving distance | 0.2912 | [0.026, 0.208] | 0.8405 | 0.7757 | 0.8677 | 0.7859 | 0.8305 |
| $W$ | AIC | adjusted $R^2$ | | | | | |
| | | Greece | Ireland | Italy | Netherlands | Portugal | Spain |
| Geographic distance | 8731 | 0.9006 | 0.7950 | 0.8345 | 0.8041 | 0.7698 | 0.8296 |
| Driving distance | 8730 | 0.9006 | 0.7951 | 0.8346 | 0.8042 | 0.7699 | 0.8296 |

**Table 7 Robustness check: the estimation and testing results under different definitions of spatial weight matrix $W$ for the S-APT Model 4s for the eurozone stock indices returns.**

| Definition of $g_{credit}$ | p-value for testing $\bar{\alpha}_0 = 0$ | C.I. of $\rho_0$ | adjusted $R^2$ | | | | |
|---|---|---|---|---|---|---|---|
| | | | Austria | Belgium | Finland | France | Germany |
| 1 | 0.2912 | [0.026, 0.208] | 0.8405 | 0.7757 | 0.8677 | 0.7859 | 0.8305 |
| 2 | 0.0652 | [0.117, 0.287] | 0.8537 | 0.7942 | 0.8786 | 0.8036 | 0.8445 |
| 3 | 0.1549 | [0.025, 0.203] | 0.8593 | 0.8021 | 0.8833 | 0.8111 | 0.8504 |
| 4 | 0.1426 | [0.009, 0.189] | 0.8535 | 0.7940 | 0.8785 | 0.8034 | 0.8443 |
| Definition of $g_{credit}$ | AIC | adjusted $R^2$ | | | | | |
| | | Greece | Ireland | Italy | Netherlands | Portugal | Spain |
| 1 | 8730 | 0.9006 | 0.7951 | 0.8346 | 0.8042 | 0.7699 | 0.8296 |
| 2 | 8585 | 0.9088 | 0.8120 | 0.8482 | 0.8203 | 0.7889 | 0.8437 |
| 3 | 8515 | 0.9123 | 0.8193 | 0.8541 | 0.8272 | 0.7970 | 0.8497 |
| 4 | 8582 | 0.9087 | 0.8119 | 0.8481 | 0.8202 | 0.7887 | 0.8435 |

**Table 8 Robustness check for empirical results of European stock index returns: the estimation and testing results under four different definitions of the credit factor for the S-APT model 4s. The spatial weight matrix $W$ is defined by driving distance. There does not seem to be significant differences in the empirical results.**

the states where the 10 MSAs are located during 2006–2013. It can be seen that the credit rating of California is significantly worse than the other states, while Florida and Nevada can be chosen as representatives of states of good credit quality as both of them have a rating as good as AA+ for at least five years during the period. Following the approach of Fama and French (1993), we construct a factor related to credit risk as the difference of futures returns of MSAs in the states with relatively bad credit and those with relatively good credit,

$$g_{credit} := r_{LosAngeles} + r_{SanDiego} + r_{SanFrancisco} - (r_{Miami} + r_{LasVegas}), \tag{44}$$

where $r_{MSA}$ denotes the return of CSI index futures of a particular MSA. In addition to the credit factor, we consider two other factors: (i) $g_{CS10f}$: the monthly return of futures on S&P/Case-Shiller composite 10-City Index, which reflects the overall national residential real estate market in the United States. (ii) $g_{CS10fTr}$: the trend factor of $g_{CS10f}$. $g_{CS10fTr}$ in the $k$th month is the difference between $g_{CS10f}$ in the $k$th month and the previous 12-month average of $g_{CS10f}$.[16] The trend factor $g_{CS10fTr}$ is inspired by a similar creation in Duan, Sun, and Wang (2012) and can be related to the notion of momentum captured by the momentum factor in Fama and French (2012). The trend factor describes the intertemporal momentum while the momentum factor focuses more on cross-sectional differences.

### 7.2. Empirical Performance of S-APT and APT Models

We estimate and test three models using the same three factors defined above: One is the S-APT model specified in (34), which assumes homogeneous variance for residuals, and the other two are APT models, which are specified with homogeneous and heterogeneous residual variances respectively. APT models with heterogeneous variances seem to be common in existing theoretical and empirical works; see Gibbons, Ross, and Shanken (1989) and Fama and French (1993, 2012), among others. In the S-APT model, the spatial weight matrix $W$ is specified based on driving distances in the same way as in Section 6.2.

We carry out the model fitting and the no asymptotic arbitrage (i.e., zero-intercept) test for the APT and S-APT models. The zero-intercept test for the APT models is specified in (3), and that for the S-APT models is specified in (40). Table 9 shows the estimation and testing results for the three models. The domain of $\rho_0$ in the conditional MLE estimation for the S-APT model is $[-2.0334, 1]$. First, in testing the zero-intercept hypothesis, the S-APT model is not rejected. Second, the S-APT model outperforms the other two models in terms of AIC; in particular, the S-APT model achieves lower AIC than the APT model with homogeneous variances. Hence, incorporating spatial interaction improves the description of the comovements of futures returns. Third, for the S-APT model, $\rho_0$ is found to be significantly positive. Fourth, there seems to be no spatial correlation among the residuals of the S-APT model, but there is significantly positive spatial correlation among the residuals of the two APT models. The estimates of the parameters of the S-APT model are reported in E-Companion EC.5.

Figure 3a shows the sample adjusted $R^2$ of the S-APT model with the zero-intercept constraint $\bar{\alpha}_0 = 0$. All the sample adjusted $R^2$ are positive except that of New York, which is $-0.39$. The negative adjusted $R^2$ may be due to the fact that the CSI Index of New York does not reflect the

[16] When $k \leq 12$, the trend factor for the $k$th month is defined as the difference of $g_{CS10f}$ in the $k$th month and the average of $g_{CS10f}$ in the previous $k-1$ months, except for the first month ($k=1$) where the trend factor is set to be zero.

| Model | S-APT | APT (heterogeneous variance) | APT (homogeneous variances) |
|---|---|---|---|
| p-value | 0.1023 | 0.2095 | 0.0659 |
| AIC | 3048 | 4023 | 4034 |
| Number of parameters | 42 | 50 | 41 |
| 95% C.I. of $\rho_0$ | [0.29, 0.45] | - | - |
| 95% C.I. of $\kappa$ for residuals | [-0.10, 0.10] | [0.30, 0.45] | [0.30, 0.45] |

**Table 9 The p-value for testing the no asymptotic arbitrage (i.e., zero-intercept) constraint, Akaike information criterion (AIC), the number of parameters, the 95% confidence interval (C.I.) of $\rho_0$ (only for S-APT), and the 95% C.I. of $\kappa$ defined in (2) for the residuals of the three models.**

overall real estate market in that area, as it takes into account only single-family home prices but not co-op or condominium prices; however, sales of co-ops and condominiums account for 98% of Manhattan's non-rental properties.[17] Therefore, we exclude the CSI Indices futures of New York from the analysis and test the S-APT on the remaining nine CSI Indices futures. The p-value of the test is 0.10, and hence the S-APT model is not rejected. The sample adjusted $R^2$ of fitting the remaining nine CSI Indices futures with the S-APT zero-intercept constraint is shown in Figure 3b; all the nine futures have positive adjusted $R^2$. $\rho_0$ is estimated to be 0.38 with the 95% confidence interval $[0.30, 0.46]$, which is significantly positive (the domain of $\rho_0$ and $\kappa$ in the conditional MLE estimation is $[-1.9897, 1]$).

We also perform diagnostics of residuals of the S-APT model. (1) Table 10 shows the correlation matrix of the home price indices futures returns $\tilde{y}_t$ in (34) and that of the residuals $\tilde{\epsilon}_t$ in (34) for the S-APT model. It seems that the S-APT model is effective in *eliminating cross-sectional correlations*. (2) Table 11 shows the sample autocorrelation functions (ACFs) of $\tilde{y}_t$ and that of $\tilde{\epsilon}_t$ of the S-APT model. It seems that the S-APT model is effective in *reducing the autocorrelations* in the data. (3) Table 12 shows the p-values of Kolmogorov-Smirnov test for normal distribution of the residuals $\tilde{\epsilon}_t$ of the S-APT model. The hypothesis that the residuals have normal distributions is rejected for none of the residuals.

[17] We try to alleviate the problem by including a condominium index return factor but this does not improve the fitting results much. As there are no futures contracts on the S&P/Case-Shiller Condominium Index of New York, we construct a mimicking portfolio of the excess return of the Condominium Index using the linear projection of the Condominium Index excess return on the payoff space spanned by the ten CSI Indices futures returns. Then, the payoff of the mimicking portfolio is defined as an additional factor. However, the sample adjusted $R^2$ of the linear projection is merely 17%, indicating that the mimicking portfolio payoff may not be a good approximation to the Condominium Index excess return.

| | Los Angeles | San Diego | San Francisco | Denver | Washington DC | Miami | Chicago | Boston | Las Vegas | New York |
|---|---|---|---|---|---|---|---|---|---|---|
| Los Angeles | 1.0000 | | | | | | | | | |
| San Diego | $0.5666^{**}$ | 1.0000 | | | | | | | | |
| San Francisco | $0.5881^{**}$ | $0.5812^{**}$ | 1.0000 | | | | | | | |
| Denver | $0.5278^{**}$ | $0.2272^{*}$ | $0.4189^{**}$ | 1.0000 | | | | | | |
| Washington DC | $0.5757^{**}$ | $0.3179^{**}$ | $0.5428^{**}$ | $0.5943^{**}$ | 1.0000 | | | | | |
| Miami | $0.4210^{**}$ | $0.4217^{**}$ | $0.5342^{**}$ | $0.2414^{*}$ | $0.2059^{*}$ | 1.0000 | | | | |
| Chicago | $0.2918^{**}$ | 0.0524 | $0.3791^{**}$ | $0.3736^{**}$ | $0.3283^{**}$ | $0.4541^{**}$ | 1.0000 | | | |
| Boston | $0.3152^{**}$ | $0.3011^{**}$ | $0.3801^{**}$ | $0.2877^{**}$ | $0.4573^{**}$ | $0.2443^{*}$ | $0.4839^{**}$ | 1.0000 | | |
| Las Vegas | 0.0953 | 0.1050 | $0.2655^{*}$ | $0.2569^{*}$ | -0.0072 | $0.3353^{**}$ | $0.2089^{*}$ | 0.0233 | 1.0000 | |
| New York | $0.3133^{**}$ | 0.1928 | $0.3118^{**}$ | 0.2035 | $0.3068^{**}$ | $0.3242^{**}$ | $0.4732^{**}$ | $0.4514^{**}$ | -0.0322 | 1.0000 |

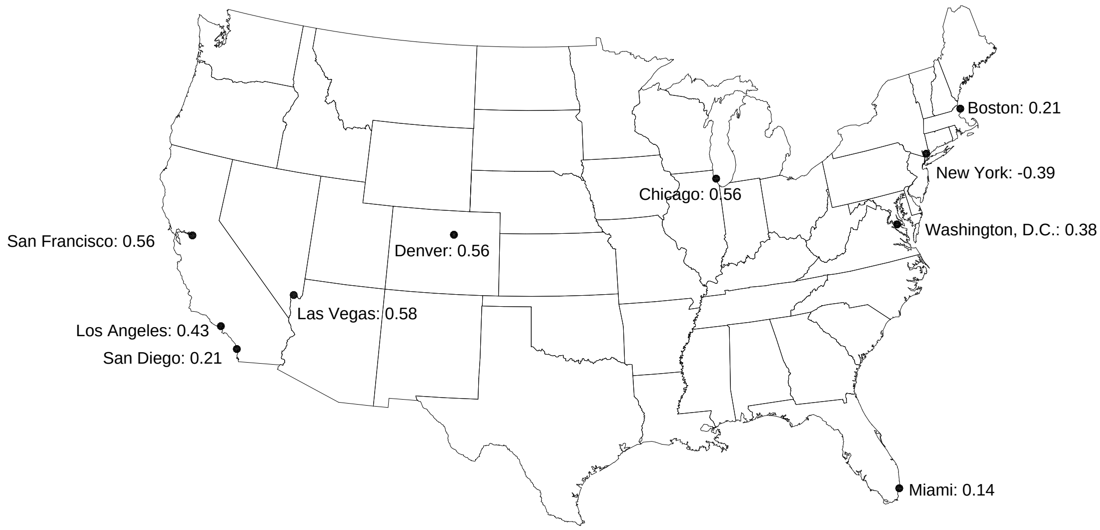
| | Los Angeles | San Diego | San Francisco | Denver | Washington DC | Miami | Chicago | Boston | Las Vegas | New York |
|---|---|---|---|---|---|---|---|---|---|---|
| Los Angeles | 1.0000 | | | | | | | | | |
| San Diego | $-0.4127^{**}$ | 1.0000 | | | | | | | | |
| San Francisco | $-0.2532^{**}$ | 0.0308 | 1.0000 | | | | | | | |
| Denver | $0.2770^{**}$ | $-0.2416^{*}$ | -0.0446 | 1.0000 | | | | | | |
| Washington DC | 0.1455 | $-0.2884^{**}$ | 0.0475 | $0.4059^{**}$ | 1.0000 | | | | | |
| Miami | 0.2076 | $0.3773^{**}$ | $0.3794^{**}$ | -0.1537 | -0.1998 | 1.0000 | | | | |
| Chicago | 0.0933 | -0.1847 | 0.0962 | 0.0481 | -0.0482 | 0.0653 | 1.0000 | | | |
| Boston | -0.1439 | 0.0860 | -0.0263 | -0.0555 | 0.0175 | -0.1279 | 0.1989 | 1.0000 | | |
| Las Vegas | 0.2025 | 0.2706 | $0.5012^{**}$ | 0.1645 | 0.0741 | 0.0904 | -0.0687 | -0.0373 | 1.0000 | |
| New York | -0.0470 | -0.0273 | -0.1424 | $-0.2148^{*}$ | $-0.3365^{**}$ | -0.0159 | 0.0789 | -0.0964 | $-0.2467^{*}$ | 1.0000 |

**Table 10 The top matrix is the sample correlation matrix of home price indices futures returns $\tilde{y}_t$ in (34); the bottom one is that of the residuals $\tilde{\epsilon}_t$ in (34) for the S-APT model. The $95\%$ significance is marked by $*$ while $99\%$ $**$. It seems that the S-APT model is effective in eliminating cross-sectional correlations.**

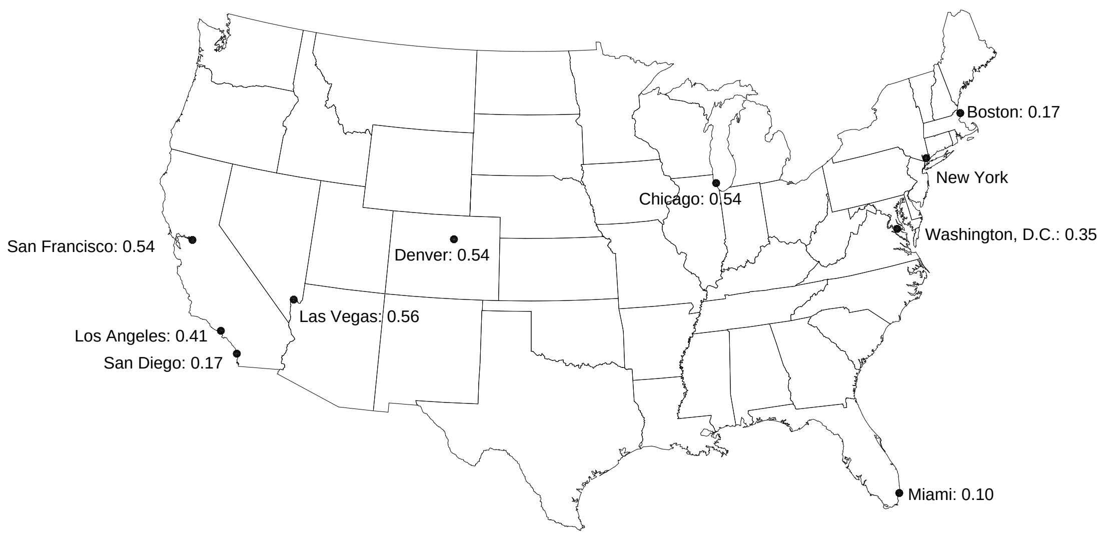
| | Lag-1 | Lag-2 | Lag-3 | Lag-4 | Lag-5 |
|---|---|---|---|---|---|
| Los Angeles | $0.4796^{**}$ | $0.2223^{*}$ | $0.2049^{*}$ | $0.2190^{**}$ | $0.2550^{**}$ |
| San Diego | $0.3647^{**}$ | $0.2391^{**}$ | 0.1812 | $0.2208^{**}$ | 0.1913 |
| San Francisco | $0.3498^{**}$ | 0.1462 | 0.0828 | 0.1045 | 0.0156 |
| Denver | $0.2825^{**}$ | 0.0634 | -0.0468 | -0.0476 | -0.0866 |
| Washington DC | $0.4082^{**}$ | 0.1241 | 0.1345 | 0.1452 | 0.1370 |
| Miami | $0.4774^{**}$ | $0.3413^{**}$ | $0.2772^{**}$ | $0.2399^{*}$ | $0.2156^{*}$ |
| Chicago | $0.4531^{**}$ | 0.1985 | 0.0027 | $-0.1832$ | $-0.3318^{**}$ |
| Boston | $0.3004^{**}$ | -0.0125 | -0.0853 | -0.0828 | $-0.2338^{*}$ |
| Las Vegas | $0.2615^{*}$ | $0.3319^{**}$ | 0.1071 | 0.1149 | 0.1799 |
| New York | $0.2882^{**}$ | 0.0638 | -0.0441 | 0.0131 | $-0.1706$ |

| | Lag-1 | Lag-2 | Lag-3 | Lag-4 | Lag-5 |
|---|---|---|---|---|---|
| Los Angeles | 0.1640 | 0.0316 | 0.0125 | -0.1104 | $-0.1455$ |
| San Diego | 0.1296 | 0.1165 | 0.0091 | 0.0375 | -0.0993 |
| San Francisco | 0.0387 | -0.1015 | 0.1234 | -0.0049 | $-0.1963$ |
| Denver | 0.1977 | -0.0198 | $-0.1967$ | -0.0907 | -0.0370 |
| Washington DC | 0.0512 | -0.1048 | -0.0258 | -0.0437 | 0.0788 |
| Miami | $0.4947^{**}$ | $0.3094^{**}$ | $0.2330^{*}$ | 0.1897 | $0.2595^{*}$ |
| Chicago | 0.0407 | -0.0239 | 0.0449 | 0.0498 | $-0.1704$ |
| Boston | 0.1742 | -0.0090 | 0.0610 | -0.0009 | 0.1232 |
| Las Vegas | 0.2096 | $0.2143^{*}$ | -0.0646 | -0.0632 | 0.0283 |
| New York | $0.2909^{**}$ | 0.1077 | -0.0099 | 0.0994 | -0.1423 |

**Table 11 The top matrix shows the sample autocorrelation functions (ACFs) for lags up to 5 of home price indices futures returns $\tilde{y}_t$ in (34); the bottom one shows those of the residuals $\tilde{\epsilon}_t$ in (34) for the S-APT model. The $95\%$ significance is marked by $*$ while $99\%$ is marked by $**$. It seems that the S-APT model is effective in reducing the autocorrelations in the data.**




(a)




(b)

**Figure 3 (a) shows the adjusted $R^2$ of fitting the three-factor model (34) with the S-APT zero-intercept constraint $\bar{\alpha}_0 = 0$ to the 10 CSI Indices futures returns. (b) shows the adjusted $R^2$ of the same model fitting as in (a), except that New York is excluded from the analysis. In the model fitting, $W$ is specified using driving distances.**

| City | Los Angeles | San Diego | San Francisco | Denver | Washington DC | Miami | Chicago | Boston | Las Vegas | New York |
|---|---|---|---|---|---|---|---|---|---|---|
| p value | 0.4614 | 0.5159 | 0.5335 | 0.0772 | 0.1533 | 0.4049 | 0.7698 | 0.9164 | 0.1193 | 0.8592 |

**Table 12 P-values of Kolmogorov-Smirnov test for the normal distribution of the residuals $\tilde{\epsilon}_t$ in (34) for the S-APT model for the home price indices futures returns. The hypothesis that the residuals have normal distributions is rejected for none of the futures returns.**

### 7.3. Robustness Check

The empirical results reported above seem to be robust with respect to different specifications of spatial matrix $W$. Table 13 compares the model testing and estimation results for $W$ defined by geographic distance and driving distance. The table shows that the results are robust to the specification of $W$. The numerical values of $W$ can be found in E-Companion EC.5.

(a) First robustness check: different $W$ for ten MSAs including New York.

| $W$ | p-value for testing $\bar{\alpha}_0 = 0$ | C.I. of $\rho_0$ | AIC | adjusted $R^2$ | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | Los Angeles | San Diego | San Francisco | Denver | Las Vegas |
| Geographic distance | 0.1008 | [0.2975, 0.4505] | 3045 | 0.4335 | 0.2119 | 0.5611 | 0.5604 | 0.5795 |
| Driving distance | 0.1023 | [0.2910, 0.4450] | 3048 | 0.4332 | 0.2088 | 0.5593 | 0.5586 | 0.5778 |
| $W$ | | | | adjusted $R^2$ | | | | |
| | | | | Washington D.C. | Miami | Chicago | Boston | New York |
| Geographic distance | | | | 0.3853 | 0.1430 | 0.5619 | 0.2101 | -0.3866 |
| Driving distance | | | | 0.3828 | 0.1396 | 0.5602 | 0.2070 | -0.3921 |

(b) Second robustness check: different $W$ for nine MSAs excluding New York.

| $W$ | p-value for testing $\bar{\alpha}_0 = 0$ | C.I. of $\rho_0$ | AIC | adjusted $R^2$ | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | Los Angeles | San Diego | San Francisco | Denver | Las Vegas |
| Geographic distance | 0.1014 | [0.3074, 0.4626] | 2780 | 0.4095 | 0.1757 | 0.5409 | 0.5401 | 0.5601 |
| Driving distance | 0.1018 | [0.3009, 0.4571] | 2783 | 0.4070 | 0.1722 | 0.5390 | 0.5382 | 0.5583 |
| $W$ | | | | adjusted $R^2$ | | | | |
| | | | | Washington D.C. | Miami | Chicago | Boston | |
| Geographic distance | | | | 0.3570 | 0.1036 | 0.5417 | 0.1738 | |
| Driving distance | | | | 0.3543 | 0.0988 | 0.5398 | 0.1704 | |

**Table 13 Robustness check of the empirical results for different definition of $W$.**

The empirical results reported above also seem to be robust with respect to the specifications of the credit factor. We consider the following four alternative definitions of the credit factor:

- Definition 1: $g_{credit} := r_{LosAngeles} + r_{SanDiego} + r_{SanFrancisco} - (r_{Miami} + r_{LasVegas})$ (the definition used in Section 7.2)
- Definition 2: $g_{credit} := r_{LosAngeles} + r_{SanDiego} + r_{SanFrancisco} - r_{LasVegas}$
- Definition 3: $g_{credit} := r_{LosAngeles} + r_{SanFrancisco} - (r_{Miami} + r_{LasVegas})$
- Definition 4: $g_{credit} := r_{LosAngeles} + r_{SanFrancisco} - r_{LasVegas}$

Table 14 shows that the estimation and testing results for fitting the S-APT model to the home price indices futures returns under the four definitions of credit factor are similar.

| Definition of $g_{credit}$ | p-value for testing $\bar{\alpha}_0 = 0$ | C.I. of $\rho_0$ | AIC | adjusted $R^2$ | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | Los Angeles | San Diego | San Francisco | Denver | Las Vegas |
| 1 | 0.1023 | [0.2910, 0.4450] | 3048 | 0.4332 | 0.2088 | 0.5593 | 0.5586 | 0.5778 |
| 2 | 0.0584 | [0.2309, 0.3951] | 3058 | 0.4205 | 0.1911 | 0.5494 | 0.5487 | 0.5683 |
| 3 | 0.0640 | [0.3381, 0.4839] | 3039 | 0.4427 | 0.2221 | 0.5667 | 0.5660 | 0.5849 |
| 4 | 0.0814 | [0.2908, 0.4452] | 3048 | 0.4326 | 0.2079 | 0.5588 | 0.5581 | 0.5773 |
| Definition of $g_{credit}$ | | | | adjusted $R^2$ | | | | |
| | | | | Washington D.C. | Miami | Chicago | Boston | New York |
| 1 | | | | 0.3828 | 0.1396 | 0.5602 | 0.2070 | -0.3921 |
| 2 | | | | 0.3690 | 0.1203 | 0.5503 | 0.1892 | -0.4233 |
| 3 | | | | 0.3932 | 0.1540 | 0.5675 | 0.2203 | -0.3687 |
| 4 | | | | 0.3822 | 0.1387 | 0.5597 | 0.2062 | -0.3936 |

**Table 14 Robustness check for empirical results of home price indices futures returns: the estimation and testing results under different definitions of the credit factor. The spatial weight matrix is derived by driving distance. There does not seem to be significant differences in the empirical results.**

# 8. Conclusion

Although there are growing evidences that spatial interaction plays a significant role in determining prices and returns in both stock markets and real estate markets, there is as yet little work that builds explicit economic models to study the effects of spatial interaction on asset returns. In this paper, we add to the literature by studying how spatial interaction affects the risk-return relationship of financial assets. To do this, we first propose new asset pricing models that incorporate spatial interaction, i.e., the spatial capital asset pricing model (S-CAPM) and the spatial arbitrage pricing theory (S-APT), which extend the classical asset pricing theory of CAPM and APT respectively. The S-CAPM and S-APT explicitly characterize the effect of spatial interaction on the expected returns of both ordinary assets and future contracts. Next, we carry out empirical studies on the eurozone stock indices and the futures contracts on S&P/Case-Shiller Home Price Indices using the S-APT model. Our empirical results suggest that spatial interaction is not only present but also important in explaining comovements of asset returns.

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