Bond Liquidity Premia

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Introduction 1

- Recent models highlight the dual role of intermediaries as arbitrageurs and liquidity providers (Brunnermeier and Pedersen, 2008).
- Lower wealth hinders their ability to pursue arbitrage opportunities and provide liquidity across markets.
- Variations in the funding costs of financial intermediaries drive commonality in valuation (risk premia) and liquidity across markets .
- We live between two extremes where financial intermediaries are infinitely capitalized and can borrow to absorb any order imbalances or, in contrast, where they must fund every position out of their own capital.

But how can we measure this funding liquidity component?

Introduction 2

- One needs to distinguish between Market Liquidity and Funding Liquidity.
    - Asset's Market Liquidity: the ease with which it is traded.
    - Traders' Funding Liquidity: the ease with which they obtain funding.
- Liquidity Risk:
    - Market Liquidity Risk: The risk that the market liquidity worsens when one needs to unwind a position.
    - Funding Liquidity Risk: The risk that a trader cannot fund his position and is forced to unwind.

Introduction 3

- Market Liquidity is low when it is difficult to raise money by selling an asset, that is when selling depresses the sale price.
- Three forms of market liquidity:
  - Bid-ask spread: how much a trader can lose by selling an asset and buying it back right away.
  - Market depth: how many units traders can sell or buy at the current bid or ask price without moving the price
  - Market resiliency: how long it takes for prices that have fallen to bounce back.

Introduction 4

- Funding liquidity is high when it is easy to borrow money to purchase assets. Margin lending is short term since margins can be adapted to market conditions on a daily basis.
- Three forms of funding liquidity:
  - Margin funding risk: risk that margins will change.
  - Rollover risk: risk that it will be more costly or impossible to roll over short-term borrowing.
  - Redemption risk: risk that demand depositors of banks or say equity holders of hedge funds withdraw funds.

Introduction 5

- In the US, changes in dealer repurchase agreements (repos) is the primary margin of adjustment of the aggregate balance sheets of intermediaries (Adrian and Shin, 2008). These are the main players in this market.
- Repo rates and, hence, funding costs, are lower for recently issued (i.e. on-the-run) than for seasoned Treasury securities. This induces the on-the-run premium.
- Funding liquidity is also lower. This contributes to the on-the-run premium.
- The on-the-run premium captures anticipations of future conditions on funding markets (price and liquidity) but it also captures its current valuation.

Introduction 6

- We measure the value of funding liquidity from the cross-section of on-the-run premia across maturities.
- We use an arbitrage-free term structure model to correct for maturity and coupon differences but add a liquidity factor to capture the difference between on-the-run and off-the-run issues.
- This measure is not driven by any other risk factors.
- Our focus on a slow-moving (monthly frequency) common component captures the valuation component of on-the-run premia.

Results - Risk Premia

- We find that liquidity value affects the cross-section of risk premia at quarterly and annual horizons.
- An increase in the value of liquidity predicts
  1. lower risk premia for on-the-run *and* off-the-run bonds,
  2. higher risk premia on LIBOR loans,
  3. higher risk premia on swap contracts,
  4. higher risk premia on corporate bonds.
- The measured impact is pervasive through crisis (flight to quality) and normal times.

This accords with the proposition that the pricing kernel in these markets includes the shadow cost of capital that intermediaries face.

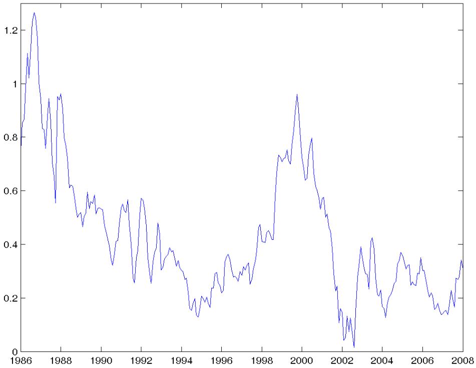
Results - Economic Drivers

We find that liquidity value varies with :

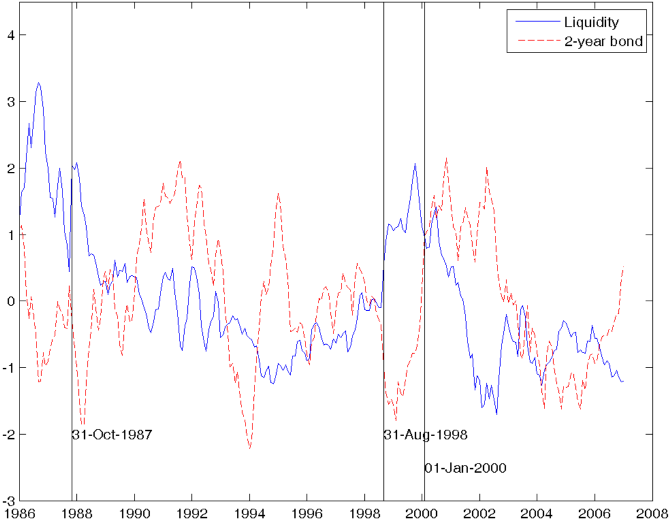
1. Changes in monetary conditions, measured from bank reserves and monetary aggregates.
2. Changes in aggregate uncertainty, measured from SP500 options.
3. Changes of bid-ask spreads on the US Treasury market.

The link between these measures and the valuation of on-the-run relative to off-the-run bonds points to the key role of funding constraints for asset valuation and market liquidity.

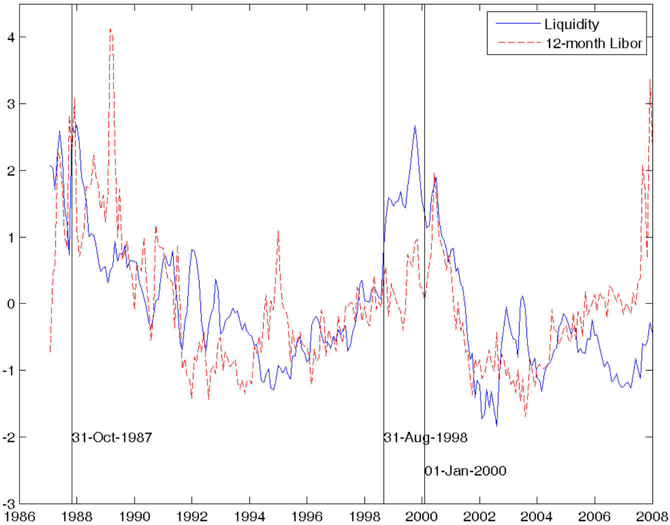
Liquidity factor: impact on a just-issued 10-year government coupon bond (scale in $)



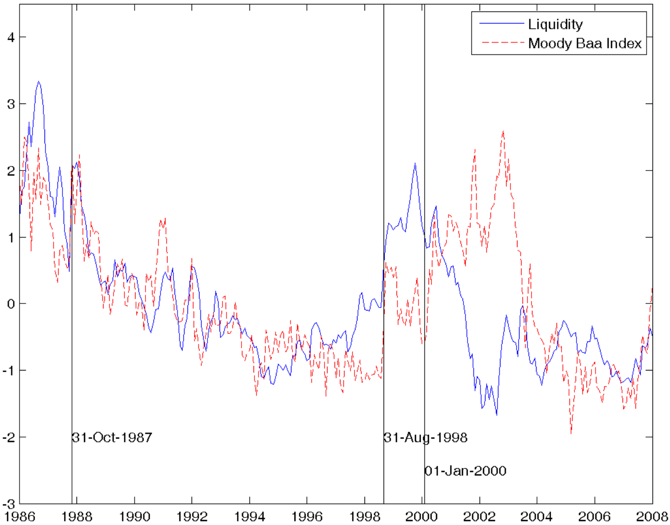
Liquidity and off-the run excess returns



Liquidity and 12-month Libor spread



Liquidity and Moody's corporate spread index



On-the-run Premium in Practice

- October 15, 1998: surprise cut in target overnight Federal Funds rate.

  > *"Greenspan said that what convinced him he had to move was the gap that had opened between the price of the current "on-the-run" 30-year U.S. Treasury bond and the price of the bond issued the year before."*

  Martin Mayer, *The Fed*, 2001.

- Also, minutes from the meeting reveal the emphasis on the co-movements of on-the-run premia, swap spreads and the spreads of mortgage-backed securities.

Funding in the Repo market

- There is both empirical and theoretical support for the link between the repo market and the on-the-run premium, whereby the most recently issued (on-the-run) bonds sell at a premium relative to seasoned (off-the-run) bonds with similar coupons and maturities.
- The on-the-run premium reflects the willingness to hold the asset that can be funded at the lowest cost and greatest liquidity to meet uncertain cash outflows and margin demands.
- As funding is fungible, this suggests that we measure the value of funding liquidity from the common component of on-the-run premia across maturities.
- Note that, in contrast with other markets, no other risk factors enter the price difference between Treasury bonds with similar coupons and maturities.

Coupon Bonds Data

- Term Structure models are mostly estimated with Fama-Bliss bootstrapped yields. This data set excludes bonds with large price difference relative to neighbors.
- We use end-of-month prices of U.S. Treasury Bills, Notes and Bonds (CRSP).
- Focus on the period from December 1985 to December 2007. Exclude issues with tax-advantages and obvious errors.

Coupon Bonds Data

- Maturity categories:
  3, 6, 9, 12, 18, 24, 36, 48, 60, 84, 120 months.
- Pick the most recent issue in category.
- Pick the issue with remaining maturity closest to category.

Coupon Bonds Data

Summary Statistics :
Mean and (standard deviation) of the difference between old and new issues in each maturity category

Maturity	Duration		Coupon		Age	
3	1.36	(0.09)			-10.40	(9.35)
6	-0.10	(0.17)			-16.84	(6.33)
9	1.12	(0.40)			-10.06	(7.29)
12	0.36	(1.02)			-10.65	(7.08)
18	2.04	(0.64)	0.02	(0.98)	-3.73	(5.98
24	0.12	(0.66)	-0.31	(2.22)	-22.42	(13.42)
36	0.07	(2.42)	-0.39	(2.07)	-20.37	(11.90)
48	2.12	(2.95)	-0.11	(1.91)	-14.26	(11.47)
60	0.94	(2.30)	-0.60	(2.05)	-26.96	(21.53)
84	2.74	(5.12)	-0.20	(2.08)	-22.25	(13.16)
120	1.04	(5.14)	0.31	(1.20)	-11.13	(17.45)

On average, the new issue has a slightly longer duration (months) and a lower coupon (%). However age differences (months) are large and highly variable.

Term Structure Model - I

Arbitrage-free dynamic extension of Nelson-Siegel:
(Christensen, Diebold and Rudebusch ,2008)

$$y_t^{(m)} = F_{1,t}\beta_1(m) + F_{2,t}\beta_2(m) + F_{3,t}\beta_3(m) \quad (1)$$

Reason to choose this model :

- Parsimonious
- Accurate in-sample and delivers better forecast
- Arbitrage-free : only differences in cash flows can drive term structure factors, F_t.
- Usual interpretation in terms of level, slope and curvature.

Term Structure Model - II

Price of coupon bond with maturity M:

$$P_t(M) = \left[\sum_{m=1}^{M} D_t(m) \times C_t(m, M)\right] + L(t, M, age) \qquad (2)$$

$$D_t(m) = \exp(-m\, y_t^{(m)})$$

The Liquidity Factor

$$L(t, M, age) = L_t \times b(M) \times exp(-\kappa \times age) \qquad (3)$$

- Common dynamic across maturities : L_t.
- Varying scale across maturities : $b(m)$.
- Varying scale across bond ages : κ.

Measurement Equation

$$P_t(M) = D_t(M)^T C_t(M) + L(t, M, age) + \nu_t(M) \quad (4)$$

$\nu_t(M)$ has standard deviation:

$$\omega(M) = \omega_0 + \omega_1 \times M \quad (5)$$

Transition Equation

$$
\begin{aligned}
(F_t - \bar{F}) &= \Phi(F_{t-1} - \bar{F}) + \Sigma\epsilon_t \\
(L_t - \bar{L}) &= \phi^l(L_{t-1} - \bar{L}) + \sigma^l\epsilon^l_t.
\end{aligned} \tag{6}
$$

- Diagonal Φ
- Lower triangular Σ

Estimation

- The state-space system is given by:

$$
\begin{aligned}
(X_t - \bar{X}) &= \Phi_X (X_{t-1} - \bar{X}) + \Sigma_X \epsilon_t \\
P_t &= \Psi(X_t, C_t, Z_t) + \Omega \nu_t,
\end{aligned}
$$

  where $X_t \equiv [F_t^T \; L_t]^T$ and Ψ is the (non-linear) mapping of cash flows C_t, bond characteristics, Z_t, and current states, X_t, into prices.

- In a linear state-space model, the Kalman recursion provides optimal estimates of current state variables given past and current prices.

Estimation

- The recursion works off estimates of state variables and their associated MSE from the previous step:

$$\hat{X}_{t+1|t} \equiv E\left[X_{t+1}|\Im_t\right],$$
$$Q_{t+1|t} \equiv E\left[(\hat{X}_{t+1|t} - X_{t+1})(\hat{X}_{t+1|t} - X_{t+1})^T\right],$$

Estimation

- The associated prediction of bond prices, and its MSE, are given by

$$\hat{P}_{t+1|t} \equiv E\left[P_{t+1}|\Im_t\right] = \Psi(\hat{X}_{t+1|t}, C_{t+1}, Z_{t+1}), \tag{7}$$

$$R_{t+1|t} \equiv E\left[(\hat{P}_{t+1|t} - P_{t+1})(\hat{P}_{t+1|t} - P_{t+1})^T\right] = \Psi(\hat{X}_{t+1|t}, C_{t+1}, Z_{t+1})^T \hat{Q}_{t+1|t} \Psi(\hat{X}_{t+1|t}, C_{t+1}, Z_{t+1}) + \Omega, \tag{8}$$

using the linearity of Ψ.

Estimation

- The next steps compare predicted to observed bond prices and update state variables and their MSEs,

$$\hat{X}_{t+1|t+1} = \hat{X}_{t+1|t} + K_{t+1}(P_{t+1} - \hat{P}_{t+1|t}), \quad (9)$$

$$Q_{t+1|t+1} = Q_{t+1|t} + K_{t+1}^{T}(R_{t+1|t})^{-1}K_{t+1}, \quad (10)$$

where

$$K_{t+1} \equiv E\left[(\hat{X}_{t+1|t} - X_{t+1})(\hat{P}_{t+1|t} - P_{t+1})^{T}\right],$$

$$= Q_{t+1|t}\Psi(\hat{X}_{t+1|t}, C_{t+1}, Z_{t+1}), \quad (11)$$

Estimation

- Finally, the transition equation gives us a conditional forecast of X_{t+2},

$$\hat{X}_{t+2|t+1} = \Phi_X \hat{X}_{t+1|t+1}, \tag{12}$$

$$Q_{t+2|t+1} = \Phi_X^T Q_{t+1|t+1} \Phi_X + \Sigma_X \Sigma_X^T. \tag{13}$$

- The recursion delivers series $\hat{P}_{t+1|t}$ and $R_{t+1|t}$ for $t = 1, \cdots, T$, from which we can construct the sample log-likelihood function:

$$L(\theta) = \sum_{t=1}^{T} l(P_t, \theta) = \sum_{t=1}^{T} \left[\log \phi(\hat{P}_{t+1|t}, R_{t+1|t}) \right], \tag{14}$$

where $\phi(\cdot, \cdot)$ is the multivariate Gaussian density.

Estimation

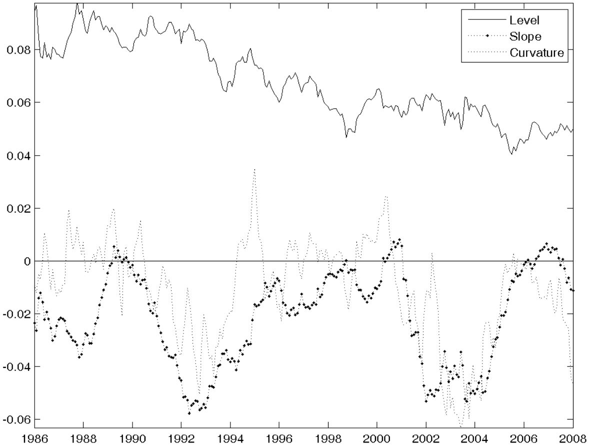
- However, because $\Psi(X, C)$ is not linear, equations (7) and (8) do not correspond to the conditional expectations of prices and the associated MSEs.
- Also, (11) does not correspond to the conditional covariance between pricing and filtering errors.
- Still, the updating equations (9) and (10) remain linear: these are justified as linear projections.
- Then we can recover the Kalman recursion provided we obtain approximations of the relevant conditional moments.

Unscented Kalman Filter

- This precisely what the UKF offers.
- The UKF is based on a method for calculating statistics of a random variable which undergoes a nonlinear transformation.
- It starts with a well-chosen set of points with given sample mean and covariance. The nonlinear function is then applied to each point and moments are computed from transformed points. Developed in the engineering literature
  Julier and Ulhman (1996)
- Recently introduced in finance
  Leipold and Wu (2003), Bakshi et al. (2005)
- Improves on linearization
  Christoffersen et al.(2007)

Estimation of the TS Model without Liquidity

- We first estimate the TS model without the liquidity factor and find results similar to CDR (2008) although we use coupon bond prices and not boostrapped sata.



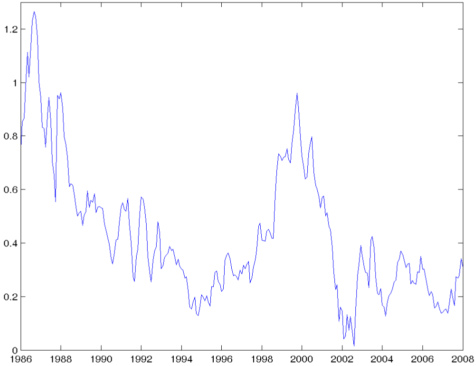
Estimation of the TS Model with Liquidity

- The model eliminates most of the systematic differences between on-the-run and off-the-run residuals.

	Residuals Differences		Liquidity Level	
Maturity	Benchmark	Liquidity	$\hat{\beta}$	QMLE s.e.
3	0.011	-0.013	0.148	(0.118)
6	0.022	-0.025	0.132	(0.118)
9	0.055	0.013	0.195	(0.129)
12	0.075	0.027	0.188	(0.114)
18	0.003	0.006	-0.074	(0.039)
24	0.028	-0.004	0.092	(0.035)
36	0.064	-0.040	0.407	(0.213)
48	0.089	-0.014	0.594	(0.359)
60	0.248	-0.035	0.965	(0.491)
84	0.124	-0.052	0.975	(0.379)
120	0.295	0.183	1	
All	0.092	0.004		

Estimation of the TS Model with Liquidity

Liquidity Factor: the value of liquidity to investors



Off-the-Run Excess Returns Regressions

Excess returns on off-the-run bonds, Term structure factors and the Liquidity factor. Data from model.

$$xr_{t+h}^{(m)} = \alpha_{m,h} + \beta_{m,h}^T F_t + \delta_{m,h} L_t + \epsilon_{t+h}^{(m)}$$

Liquidity Coefficients

	Bond Maturity									
Horizon	2		3		4		5		10	
6	-0.58	(0.32)	-0.99	(0.55)	-1.38	(0.75)	-1.74	(0.93)	-3.37	(1.70)
12	-0.47	(0.16)	-0.96	(0.31)	-1.42	(0.44)	-1.82	(0.57)	-3.50	(1.15)
18	-0.17	(0.08)	-0.50	(0.24)	-0.79	(0.38)	-1.05	(0.49)	-2.16	(0.96)
24			-0.26	(0.19)	-0.48	(0.34)	-0.68	(0.45)	-1.49	0.81)

R^2

	Bond Maturity									
Horizon	2		3		4		5		10	
6	15.1	[3.4]	15.8	[3.7]	16.7	[4.0]	17.5	[4.2]	19.2	[5.0]
12	20.4	[8.5]	22.9	[10.0]	25.7	[11.0]	28.4	[11.7]	36.0	[12.8]
18	20.7	[6.3]	19.4	[7.0]	20.5	[7.7]	22.5	[8.4]	33.6	[10.9]
24			22.2	[5.9]	18.7	[6.4]	17.1	[7.1]	20.3	[11.1]

LIBOR Spread Regressions

LIBOR Spreads, Term Structure factors and the Liquidity factor. Data from BBA and model.

$$sprd_{t+h}^{(m)} = \alpha_{m,h} + \beta_{m,h}^T F_t + \delta_{m,h} L_t + \epsilon_{t+h}^{(m)}$$

Liquidity Coefficients

Horizon	Loan Maturity 1		3		6		9		12	
0	0.073	(0.02)	0.079	(0.02)	0.068	(0.01)	0.070	(0.01)	0.069	(0.01)
6	0.045	(0.02)	0.062	(0.02)	0.067	(0.02)	0.078	(0.02)	0.081	(0.02)
12	0.091	(0.02)	0.105	(0.02)	0.103	(0.02)	0.120	(0.02)	0.131	(0.02)
18	0.088	(0.02)	0.094	(0.02)	0.094	(0.02)	0.110	(0.02)	0.118	(0.02)
24	0.109	(0.02)	0.112	(0.02)	0.113	(0.02)	0.124	(0.02)	0.130	(0.02)

R^2

Horizon	Loan Maturity 1	3	6	9	12
0	39.3	45.4	47.7	51.7	53.6
6	42.9	44.5	41.9	41.7	41.1
12	49.0	46.6	37.9	36.6	36.4
18	51.5	43.9	30.8	27.1	25.6
24	58.7	54.0	40.7	33.7	30.1

Swap Spread Regressions

Swap Spreads, Term Structure factors and the Liquidity factor. Data from Datastream and model.

$$sprd_{t+h}^{(m)} = \alpha_{m,h} + \beta_{m,h}^{T} F_t + \delta_{m,h} L_t + \epsilon_{t+h}^{(m)}$$

Liquidity Coefficients

	Contract Maturity									
Horizon	24		36		48		60		84	
0	0.068	(0.027)	0.073	(0.027)	0.076	(0.027)	0.079	(0.026)	0.094	(0.026)
6	0.072	(0.028)	0.079	(0.029)	0.086	(0.028)	0.090	(0.027)	0.111	(0.026)
12	0.094	(0.030)	0.102	(0.030)	0.111	(0.030)	0.112	(0.029)	0.137	(0.028)
18	0.140	(0.029)	0.149	(0.030)	0.152	(0.029)	0.149	(0.028)	0.166	(0.027)
24	0.111	(0.027)	0.121	(0.026)	0.125	(0.027)	0.120	(0.026)	0.128	(0.024)

R^2

	Contract Maturity									
Horizon	24		36		48		60		84	
0	20.7	[6.7]	21.1	[6.1]	21.9	[6.9]	21.9	[7.8]	24.4	[12.1]
6	12.6	[7.8]	11.7	[7.8]	13.5	[9.2]	15.0	[10.3]	20.8	[14.9]
12	14.2	[9.8]	14.8	[10.3]	17.0	[12.3]	18.2	[13.0]	23.2	[18.1]
18	22.9	[16.3]	24.5	[16.9]	25.7	[18.0]	26.0	[18.1]	29.6	[22.2]
24	18.8	[11.1]	20.9	[12.2]	21.3	[13.0]	20.7	[12.8]	20.9	[14.4]

Corporate Spread Regressions - I

Moody's Corporate Spread indices, Term Structure factors and the Liquidity factor. Data from Moody's and model.

$$sprd^{(i)}_{t+h} = \alpha_{i,h} + \beta^T_{i,h} F_t + \delta_{i,h} L_t + \epsilon^{(i)}_{t+h}$$

			Horizon							
			0		6		12		24	
Rating	AvgPremium		Liquidity Coefficients							
Aaa	2.214	(0.041)	0.154	(0.032)	0.128	(0.045)	0.141	(0.049)	0.166	(0.054)
Baa	3.314	(0.056)	0.999	(0.140)	0.787	(0.192)	0.833	(0.164)	0.470	(0.207)
			R^2							
Aaa			27.8	[18.6]	20.7	[16.1]	21.3	[19.3]	39.7	[32.2]
Baa			43.9	[26.0]	24.5	[20.6]	25.2	[24.9]	54.7	[43.3]

Corporate Spread Regressions - II

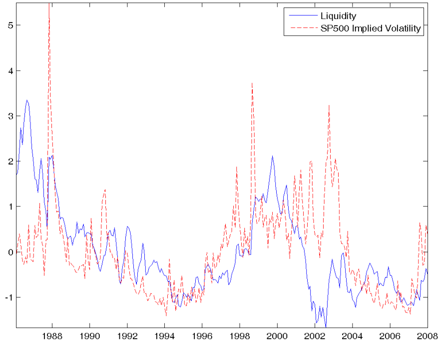
Corporate bond spreads, Term Structure factors and the Liquidity factor. Data from NAIC and model grouped in 5 rating Groups, $G = 1 \ldots 5$. Panel regression from 1996 to 2001.

$$sprd^{(i)}_{t+h} = \alpha_h + \beta_h^T X_t + \delta_{j,h} L_t \times 1(G_i = j) + \epsilon^{(i)}_{t+h}$$

			Horizon							
			0		6		12		24	
Rating	Avg Premium		Liquidity Coefficients							
G1	1.514	(0.186)	-0.389	(0.138)	-0.420	(0.148)	-0.255	(0.111)	-0.137	(0.133)
G2	1.648	(0.205)	-0.251	(0.122)	-0.314	(0.118)	-0.158	(0.097)	-0.050	(0.107)
G3	2.251	(0.299)	-0.076	(0.157)	-0.099	(0.168)	0.123	(0.136)	0.309	(0.176)
G4	3.383	(0.586)	0.234	(0.121)	0.293	(0.134)	0.420	(0.099)	0.566	(0.119)
G5	3.703	(0.536)	0.304	(0.121)	0.436	(0.143)	0.647	(0.154)	0.861	(0.149)
			R^2							
All			6.49	[2.4]	6.11	[4.1]	7.01	[4.8]	9.15	[7.1]

Aggregate Uncertainty and the Liquidity factor

- Measure uncertainty from SP500 option implied volatility.



The value of liquidity on funding markets rises with uncertainty.

Macroeconomic Factors and the Liquidity factor

- Macro factors are from Ludvigson and Ng: F2 (Financial), F4 (Inflation), F5 (Housing), F6 (Monetary Base), F7 (ST Spreads above overnight FF rate), F8 (Equities).
- BA : Difference between the median and minimum BA spreads. Measure the transaction cost difference between the most liquid and a typical bond.

Regressors										
BA	*VXO*	*F1*	*F2*	*F3*	*F4*	*F5*	*F6*	*F7*	*F8*	R^2
		0.037 (0.027)	0.089 (0.021)	-0.008 (0.020)	0.055 (0.025)	0.043 (0.021)	-0.074 (0.025)	0.082 (0.026)	-0.019 (0.019)	30.4
0.140 (0.037)										31.7
	0.094 (0.027)									14.2
0.102 (0.035)	0.031 (0.031)	0.013 (0.022)	0.060 (0.021)	-0.010 (0.016)	0.025 (0.019)	0.019 (0.019)	-0.056 (0.025)	0.070 (0.024)	-0.048 (0.016)	45.2

Conclusion

- There is a common component of on-the-run premia across maturities : it measures the marginal value of on-the-run securities as liquid collateral.
- Prices of US Treasury securities increase (risk premia decrease) when the liquidity factor increases. Their value as liquid collateral also rises.
- The liquidity factor is a risk factor in several financial markets : risk premia increases when the liquidity factor increases. This reflects the rising shadow costs of capital faced by intermediaries on the funding market.
- The liquidity factor is explained by macroeconomic factors : it rises when uncertainty increases, when monetary conditions are tighter and when transaction costs are higher.