NUS RMI Working Paper Series – No. 2019-09

Variance Risk Premium in Individual Stocks: Aggregating Factor Variance Risk

Sungjune PYUN

September, 2019

NUS Risk Management Institute
21 HENG MUI KENG TERRACE, #04-03 I[3] BUILDING, SINGAPORE 119613

www.rmi.nus.edu.sg/research/rmi-working-paper-series

Variance Risk Premium in Individual Stocks: Aggregating Factor Variance Risk

Sungjune Pyun*

National University of Singapore

This Draft: September 2019

ABSTRACT

The risk premium of stocks due to priced variance risk is summarized to two variables – the stock-specific price of variance risk (i.e., the difference between realized and option-implied variance) and the quantity of variance risk (i.e., how stock prices respond to their variance shocks). Empirically, stocks with a high negative price have higher future returns. However, this spread is significant only if stocks also have a high negative quantity. The relationship holds after controlling for market variance risk and even for non-optionable stocks, suggesting that the results are unlikely to be driven by the price pressure of informed trading.

JEL classification: G11, G12, G15 and G17.

Keywords: Variance Risk Premium, Beta Representation

*This paper, previously titled "Market and Non-market Variance Risk in Individual Stock Returns," is based on my doctoral dissertation. I thank my advisors Chris Jones, Scott Joslin, Juhani Linnainmaa, and Selale Tuzel. I also thank Zhenzhen Fan, Allaudeen Hameed, Bing Han, Yongjun Kim, Kai Li, Johan Sulaeman, Weina Zhang, and seminar participants at NUS, USC, the Australasian Finance and Banking Conference, Asia-Pacific Association of Derivatives Conference, China International Conference in Finance, and the Singapore Risk Management Conference for their helpful comments. This work was supported by NUS Start-up Grant (R-315-000-120-133). All errors are my own. Email:sjpyun@nus.edu.sg

I. Introduction

Variance risk is an essential source of risk that determines investors' inter-temporal portfolio decisions. Consistent with the premise, recent studies suggest that variance risk is priced among individual stocks. Ang, Hodrick, Xing, and Zhang (2006) suggest that stocks with high negative exposure to market variance risk have higher subsequent returns.[1] Boguth and Kuehn (2013) argue that consumption and output volatility are priced among stocks. Bansal, Kiku, Shaliastovich, and Yaron (2014) investigate how human capital volatility risk is related to asset prices. In related research, Bali, Brown, and Tang (2017) show that the common variation in macro-economic uncertainty is priced in the cross-section of stocks.

Despite the relatively well-documented importance of variance risk as a priced factor for the cross-section of stock returns, several limitations also exist. First, as high variance in one economic variable also may imply high variance in another, whether there is more than a single priced variance shock is yet unclear. The variance shocks of macroeconomic factors are cross-correlated, or one may be projections of the others. Moreover, they are also correlated with market variance shocks since the market reacts to macroeconomic shocks.[2] Hence, disentangling one from the other is empirically challenging, and using these collectively is also not straightforward.

This paper proposes a unified framework that shows how different types of priced variance risk affects individual stock returns. The framework suggests that two variables summarize the stock risk premium originating from the variance shocks of multiple sources. One is the stock-specific *price of variance risk*, roughly defined as the difference between the physical and the risk-neutral expectation of the stock return variance, and the other is the *quantity of variance risk*, which measures how stock prices react to changes in their variance shocks. This paper finds substantial evidence that variance risk, beyond those of the market, is priced

[1] See also Adrian and Rosenberg (2008), Chang, Christoffersen, and Jacobs (2013), Conrad, Dittmar, and Ghysels (2013), and Cremers, Halling, and Weinbaum (2015), among others.

[2] See for example, Officer (1973), Schwert (1989), Kandel and Stambaugh (1990), Whitelaw (1994), Bekaert, Engstrom, and Xing (2009), and Engle, Ghysels, and Sohn (2013), among others for discussions on the relation between market variance and economic variances.

among individual stocks. Stocks with a negative high price of variance risk outperform. However, the difference in performance increases for stocks that have a high negative quantity of variance risk. The difference in the spread is more than 1% per month. The difference in spreads remains both economically and statistically significant after controlling for the market variance risk in multiple ways.

The framework of this paper is motivated by the fact that the price and quantity of variance risk should jointly determine the risk premium of assets. Bekaert and Hoerova (2014) and Han and Zhou (2012) suggest that the stock-specific price of variance risk is negatively related to future stock returns. The framework of this paper implies that another dimension, the quantity of variance risk, is at least as important as the price. The price should matter only for stocks with a high negative quantity, and the quantity should matter only for stocks that have a high negative price of variance risk.

The price and quantity of variance risk have distinct roles. The price of variance risk of individual stocks is determined by the underlying factor structure of stock returns. Specifically, consider two excess returns ($R_i, i = 1, 2$) generated by the market model with different betas:

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + \epsilon_{i,t}, \tag{1}$$

where R_m is the market excess return. It naturally follows that the variance of stock returns ($V_{i,t}$) is linear in the beta squared (β_i^2).

$$V_{i,t} = \beta_i^2 V_{m,t} + V_{e,i,t}. \tag{2}$$

Therefore, a positive market variance shock increases the variance of both stocks 1 and 2, but they are unequally affected. The variance of the higher beta stock should fluctuate more, which implies higher variance risk for investors. Moreover, if the market variance has a negative price of risk, the price for the variance of the high beta stock should also be more negative.

Now consider two stocks A and B, that are similar in their market betas but have different exposures to inflation risk. Assume that only the prices of stock B react to inflation shocks. Suppose an announcement of the Fed increases uncertainty in future directions of the inflation rate, reduces uncertainty in future employment, but as a total, has little influence on the market variance. In this example, the Fed announcement will only affect the variance of stock B as stock A is unexposed to inflation risk. Also, if inflation uncertainty risk has a negative price, stock B's price of variance risk should be more negative compared to that of A. These two examples suggest that stock variances may have a unique price of risk, and the stocks exposures to risk factors determine the price.

Then following the well-known "beta representation," the quantity of variance risk (i.e., a stock's sensitivity to variance shocks) determines how the stock risk premium should be related to the price of the stock variance risk. This combination of the price and quantity constitutes a fraction of the total risk compensation given to the stock investors and captures how variance shocks are priced, regardless of the source of uncertainty. Empirically, this representation indicates that the price and the quantity of the stock variance risk should have interactive roles. That is, the prices should matter more for stocks with a large negative quantity of risk, and vice versa.

Focusing on the variance risk of individual stock returns is also useful for two practical reasons. First, the combination of the price and quantity of risk has a direct interpretation as a risk premium. That is, this combination measures the one-month risk premium of the stock that compensates for bearing various types of variance or uncertainty risk, which is not only restricted to market variance risk. Thus, the combination constitutes a fraction of the stock risk premium, of which the size of the fraction depends on the correlation between stock returns and its variance shocks. When variance shocks explain a large proportion of risk, the combination should explain a more substantial portion of the risk compensation.

Second, the estimation of both the price and the quantity of variance risk is relatively straightforward. The price of variance risk can be directly estimated by the difference between the realized variance (RV) and the option-implied variance. The underlying factor (i.e., unexpected changes of variance) is also identifiable. The quantity of variance risk can then be estimated using regressions of stock returns on variance shocks. Therefore, we can calculate both the price and the quantity of variance risk.

One of the primary interest of this paper is whether the price and quantity of variance risk are jointly priced. First, this paper confirms earlier findings that suggests a negative relationship between the price of variance risk and future stock returns. Although statistically insignificant, the quantity of variance risk and future stock returns are also slightly negative.

Most importantly, this paper confirms a robust interactive relationship between the two. During the period 1996 – 2016, the monthly return spread between stocks with a negative and positive price of variance risk is -0.75%. However, the sign and the size of the spread depend primarily on how stock returns react to their variance shocks. It is more negative (-1.35%) for stocks with a high negative quantity than for stocks with a relatively positive (-0.10%) quantity of risk. Analogously, the overall spread between the high minus low portfolio of the quantity-sorted stocks is -0.11%. However, the spread is significantly negative for stocks with a high negative price of risk (-0.91%). For the other portfolio that consists of stocks with a smaller price of variance risk, the spread is positive (0.34%). Finally, trading on the price and quantity of variance risk simultaneously yields as much as 1.25% per month in excess returns and 1.09% after adjusting for the common factors.

This paper provides two points of evidence that suggest that exposure to aggregate market variance risk (Ang, Hodrick, Xing, and Zhang 2006) has a limited role in explaining the interactive relationship. First, the price and quantity of variance risk orthogonal to market variance risk are estimated. This paper finds that they also play an interactive role in explaining future stock returns. Second, risk-adjusted returns are calculated after controlling for the market variance (FVIX) factor of Ang, Hodrick, Xing, and Zhang (2006). The relationships remain significant for risk-adjusted returns.

The interactive relationship is not only restricted to stocks that have options traded. The framework of this paper suggests that the variance of a stock is increasing in the square of factor loadings. Based on this theoretical relationship, the price can be extrapolated to all traded stocks, even to those without options. The analyses show that the interactive relationship remains for stocks without options, suggesting that the results are unlikely to be driven by price pressures induced by informed trading.

This paper contributes to several streams of literature. First, several recent studies investigate the role of stock-level variance in predicting the cross-sectional variation of stock returns. Bali and Hovakimian (2009) and Han and Zhou (2012) report a negative relationship between the price of variance risk and subsequent stock returns. Han and Zhou (2012) argue that the price of risk is a proxy for the exposure to market variance risk, while Bali and Hovakimian (2009) claim that it proxies for jump risk. Ang, Hodrick, Xing, and Zhang (2006), among others, show that exposures to market variance risk are related to future stock returns. To my best knowledge, the joint role of the price of risk and the risk exposure for individual stocks has not been analyzed, which is the focus of this paper.[3]

Second, several theoretical works study the relationship between stock-level and factor-level variance risk. For example, Boguth and Kuehn (2013) study consumption and output volatility. Bekaert, Hoerova, and Lo Duca (2013) and David and Veronesi (2014) examine the prospect that the market and uncertainty in monetary policy stance may be related. Bansal, Kiku, Shaliastovich, and Yaron (2014) investigate how human capital volatility risk is related to asset prices. This paper reconciles several of the theoretical predictions and reinforces empirical findings of this literature.

Third, several studies including Goyal and Santa-Clara (2003), Fu (2009), Chen and Petkova (2012), Stambaugh, Yu, and Yuan (2015), and Hou and Loh (2016), among others investigate the pricing implications of idiosyncratic volatility. The focus of this paper is slightly different from theirs as the subject of interest is on variance risk rather than on the level of variances. One exception is Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016), but they mainly focus on purely idiosyncratic variance risk. The main findings of this paper

[3] For the market index, Pyun (2019) argues that the price and the quantity of the market variance risk jointly determines the risk premium of the market.

are robust after controlling for the common factors of idiosyncratic volatility risk of Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016).

II. The Model

Whether the stock prices of a firm are positively affected, negatively affected, or unaffected by economic uncertainty shocks[4] depends on multiple aspects. For example, if some systematic uncertainty shocks increase uncertainty of future cash flows, they should negatively affect stock prices. Albeit generally to a lesser degree, uncertainty shocks may also affect stock price positively, if they are associated with technological innovation. For example, Pástor and Veronesi (2009) show how uncertainty shocks can lead to positive stock prices movements. Segal, Shaliastovich, and Yaron (2015) suggests that volatility shocks that have a positive and negative impact on the economy have the opposite pricing implications.

Stock prices also may react to idiosyncratic volatility shocks. A firm may possess growth options obtained from internal research or external purchases. If the growth option has a substantial value, the firm has limited downside risk and with considerable upside potential. A firm-specific uncertainty shock may increase the value of the growth option (Grullon, Lyandres, and Zhdanov 2012, Barinov 2013). Limited liability also generates option-like payoff in equity. A high variance may be preferable to equity holders if a high possibility of default exists. Firms also may be subject to political risk, ownership risk, or other types of firm-specific risk that may affect the volatility of the firm and stock prices simultaneously.

A stock investors perception of uncertainty shocks depends on the stock price reaction to these uncertainty shocks. For example, the performance of a labor-intensive firm may depend on unemployment shocks. The stock prices of this firm would also naturally respond to changes in labor uncertainty risk. If labor uncertainty shocks are bad for the firm, the stock price will react negatively; if it is good, the price will increase. Also, if investors dislike (like)

[4]Following recent studies, e.g., Bollerslev, Tauchen, and Zhou (2009), Bekaert, Engstrom, and Xing (2009), and Drechsler and Yaron (2011), among others. Variance shocks and uncertainty shocks in this paper are used inter-exchangeably.

high labor uncertainty, they should require higher compensation for stocks that negatively react to these shocks.

In this example, the price of variance risk partly measures whether the representative investor dislikes high labor uncertainty. Therefore, whether the stock investor requires compensation for bearing these uncertainty risk depends on these two dimensions. The stock investor should not require any compensation if he/she has no preference for high/low uncertainty. There should also be no compensation if stock prices are unaffected by these uncertainty shocks.

In this section, I provide a model that illustrates this point. The stock-specific price of variance risk is defined as the negative covariance between the innovations in the stock variances and the stochastic discount factor (SDF). If certain variance shocks are perceived to be unfavorable to the marginal investor, the price should be negative. Let Λ_t be the marginal utility of the representative agent with some utility function. Mathematically, stock i's price of variance risk is defined as

$$\lambda_{v,i,t} = \mathrm{Cov}_t\left(-\frac{d\Lambda_t}{\Lambda_t}, dV_{it}\right) \approx E[dV_{it}] - E^Q[dV_{it}] \tag{3}$$

where $\frac{d\Lambda_t}{\Lambda_t}$ is the SDF represented in continuous time, and V_{it} is stock i's variance at time t.[5]

Suppose that the price of stock i ($S_{i,t}$) follows a stochastic process that can be decomposed as the sum of multiple factors. The stock return can be represented as the sum of the stock-specific systematic risk ($dY_{i,t}$) and idiosyncratic risk ($dW^{idio}_{i,t}$):

$$\frac{dS_{i,t}}{S_{i,t}} = a_i dt + dY_{i,t} + \sigma_{idio} dW^{idio}_{i,t}. \tag{4}$$

In factor representation, the stock-specific systematic risk $dY_{i,t}$ can be expressed as a linear combination of multiple factors and their corresponding slopes, the betas. For example,

[5] The approximation comes from ignoring the risk-free rate. Note that this is the price of the stock variance. That is, if an asset that resembles the variance process of stock i exists, the risk premium of this specific asset is $\lambda_{v,i,t}$.

if stock i follows an N-factor structure, $dY_{i,t}$ can be represented as $\sum_{n=1}^{N} \beta_{n,i} dF_{n,t}$, where $dF_1, dF_2, \ldots, dF_N$ are N-independent factors and $\beta_{n,i}$ are the corresponding slopes. $dW_{i,t}^{idio}$ represents unpriced idiosyncratic risk, of which the variance is assumed to be constant.[6]

Let each of the risk factors $dF_{n,t}$ further follow a stochastic process that is correlated with the corresponding variance innovations. That is, let

$$
\begin{aligned}
dF_{n,t} &= \mu_{n,t} dt + \sqrt{V_{n,t}}(\rho_n dW_{n,t}^v + \sqrt{1-\rho_n^2} dW_{n,t}^o) \\
dV_{n,t} &= \theta_n dt + \sigma_{v,n} dW_{n,t}^v,
\end{aligned}
$$

where $V_{n,t}$ is the variance of factor n, and $dW_{n,t}^v$ and $dW_{n,t}^o$ are independent Brownian motions. The correlations between factor returns and variance innovations (ρ_n) are assumed to be constant but can be time varying.

In this representation, factor returns ($dF_{n,t}$) are further decomposed into two parts. The first component is related to how the factor is related to the variance of the factor and is captured by the parameter ρ_n. The sign of this parameter depends on the characteristics of the factor. For example, for the market factor, this correlation is typically negative since as investors require a higher risk premium for being in a high variance state (French, Schwert, and Stambaugh 1987).

The second component is related to what is orthogonal to variance shocks. I refer to them as orthogonal shocks. They may include one-time cash flow shocks that do not alter business risk or environment and residual macroeconomic shocks unrelated to their volatility changes. This decomposition into a variance-related and orthogonal component is always possible.

A stock's exposure to a factor variance risk is determined by both the stock's factor beta ($\beta_{n,i}$) and the factor's variance beta (i.e., how the factor returns change in response to their

[6] In the world where we can essentially have an infinite number of factors, idiosyncratic risk refers to risk that is unpriced in any form. Hence, the variance of idiosyncratic risk is unpredictable and unpriced.

variance shocks). To see this, one can solve the above three equations together and express returns as a function of factor variances. Solving the equation yields

$$\frac{dS_{i,t}}{S_{i,t}} = \{\cdot\}dt + \sum_n (\beta_{n,i}\rho_n \frac{\sqrt{V_{n,t}}}{\sigma_{v,n}})dV_{n,t} + \sum_n B_{n,i} dW^o_{i,t} + dW^{idio}_{i,t}, \tag{5}$$

where $B_{n,i} = \beta_{n,i}\sqrt{V_{n,t}(1-\rho_n^2)}$ and $\{\cdot\}$ is used for some function that can be solved explicitly but is unnecessary to derive the final formula. One can see that $\rho_n \frac{\sqrt{V_{n,t}}}{\sigma_{v,n}}$ is the slope that one would get if factor returns were regressed on their variance shocks.

Deriving the relationship between the stock risk premium and the price of factor variance risk follows the logic of Pyun (2019). Let the discounted marginal utility be Λ_t. Then, the expected excess return of a stock can be expressed as a function of the SDF as

$$\mathrm{Cov}_t(-\frac{d\Lambda_t}{\Lambda_t}, \frac{dS_{i,t}}{S_{i,t}}) = \sum_n (\beta_{n,i}\rho_n \frac{\sqrt{V_{n,t}}}{\sigma_{n,v}})\mathrm{Cov}_t(dV_{n,t}, -\frac{d\Lambda_t}{\Lambda_t}) + \sum_n B_{n,i}\mathrm{Cov}_t(dW^o_{n,t}, -\frac{d\Lambda_t}{\Lambda_t}). \tag{6}$$

The first key result of this paper is summarized as below:

Result 1. *Assume stock i follows an N-factor structure with factors $\{F_1, \ldots, F_N\}$ and corresponding betas $\{\beta_{1,i}, \ldots, \beta_{N,i}\}$. Let $\lambda_{n,t}$ be the price of variance risk of the n^{th} risk factor, $\lambda_{o,i,t}$ be the linear combination of the price of risk given orthogonal shocks*[7] *and $\beta_{v,n,i,t}$ the slope of a hypothetical regression of stock i's returns on factor ns variance shocks (dV_n). Then we have:*

$$E_t[R_{i,t+1}] = \sum_{n=1}^{N} \beta_{v,n,i,t}\lambda_{n,t} + \lambda_{o,i,t}. \tag{7}$$

Equation (7) suggests that the expected excess return of a stock is represented as a linear combination of the product of the price and quantity of variance risk of latent factors and that of the orthogonal component. However, estimating these components is not straightforward

[7]Here, $\lambda_{n,t} = \mathrm{Cov}_t(dV_{n,t}, -\frac{d\Lambda_t}{\Lambda_t})$ and $\lambda_{o,i,t} = \sum_n B_{n,i}\mathrm{Cov}_t(dW^o_{n,t}, -\frac{d\Lambda_t}{\Lambda_t})$.

since it would require assuming a specific factor structure and computing the price of variance risk of each factor separately. Determining the factors to be used is often obscure.

The second result (proof in the appendix) suggests that the first component of the representation, the risk premium that comes from systematic variance shocks, can be transformed into a combination of two variables, the price and the quantity of individual stock variance risk.

Result 2. *Let the price of variance risk of stock i to be $\lambda_{v,i,t}$, and the stock i's exposure to the variance shocks be $\beta_{v,i,t}$. Assuming that idiosyncratic volatility remains constant, we have the following alternative representation of Result 1:*

$$E[R_{i,t}] = \beta_{v,i,t}\lambda_{v,i,t} + \lambda_{o,i,t}, \tag{8}$$

where the price of stock variance risk ($\lambda_{v,i,t}$) is defined as the covariance between the negative SDF and individual stock variances, and the quantity of stock variance risk is the price sensitivity of the stock to the variance shocks. Note that $\beta_{v,i,t}$ is equivalent to the beta of stock returns regressed on stock variance innovations. The representation shows that the product of the price and quantity exactly quantifies the stock's risk premium implied by systematic variance shocks. Also, notice that the price of orthogonal risk $\lambda_{o,i,t}$ remains the same as in Result 1.

In this framework, the stock-specific price of variance risk is determined by the factor structure of the stocks and the size of the price of factor variances risk. If returns are assumed to be linear in betas, the variance process of the stock is linear in the squared betas. All else equal, if a factor carries a negative price of risk, the price of variance risk of a stock should be more negative for stocks that have a high absolute beta. For the market factor, where betas are mostly positive, the price of stock-specific variance risk should be higher for high beta stocks.

The signs and magnitudes of the quantity are determined jointly by the first-order factor exposure (the betas) and the return-variance correlations of the factors. If a factor has a negative return-variance correlation, a high positive beta stock outperforms when variance decreases. Since stock variances also decrease during these times, they are also likely to be more negatively exposed to their variance shock. Therefore, a high beta stock would have a negative quantity of variance risk. In contrasts, if the return-variance correlation of a factor is positive, a high beta stock will gain when the factor variance increases. High beta stocks should have a relatively positive quantity of variance risk. In the empirical section, I provide evidence that is consistent with this explanation for at least several of the Fama and French (2015) factors.

The framework of this paper is especially useful under a dynamic factor structure. The premise that the factor structure may be dynamic is not unrealistic. For example, evidence suggests that the variance risk exposure may be time varying. Ang, Hodrick, Xing, and Zhang (2006) and Adrian and Rosenberg (2008), among others, find that stocks with negative exposure to market variance risk tend to outperform those with positive exposure. Chang, Christoffersen, and Jacobs (2013) find weaker evidence when the betas are estimated over a longer horizon, which suggests that variance betas may be time varying.

Several remarks related to the main results are worth mentioning. First, this model assumes that idiosyncratic variance remains constant over time. One may wonder how time-varying idiosyncratic volatility affects the relation between the expected stock returns and the price of variance risk. Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016) suggest that idiosyncratic volatility is time varying and that co-movement across stocks is priced. They argue that idiosyncratic volatility may affect heterogeneous agents, as idiosyncratic volatility shock may affect income risk faced by households.

The implication of time-varying idiosyncratic variance is straightforward. In Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016), the common variation in the idiosyncratic variance is considered to proxy exposure to labor income risk. This common component can be considered as if there exists a latent factor that determines the co-moving volatility component. The relation breaks when an unpriced idiosyncratic volatility risk component varies over time. A time-varying idiosyncratic variance biases the quantity, $\beta_{v,i,t}$, towards zero, as if an error-in-variables problem arises during the estimation process. As a result, the quantity of variance risk of individual stock returns may be biased, where the size depends on the relative proportion of the volatility of the unpriced variance to the total variance of the stock.

Second, one may wonder why each stock has a unique price of risk. In this framework, the uniqueness is due to the differences in factor exposures of each stock. The price is referred to as a price of risk but is a combination of the prices of different sorts of variance risk. As described earlier, stocks that are more subject to the underlying risk are also more likely to be exposed to factor uncertainty shocks. Therefore, the cross-sectional variation of the price of variance risk mainly stems from the difference in factor exposures.

Third, one limitation of this approach is that the risk premium of the orthogonal component ($\lambda_{o,i,t}$) is essentially not estimable. As mentioned, this type of risk includes systematic shocks that do not lead to changes in the level of variance.

Finally, this paper suggests that the price and quantity of risk have joint roles in determining the risk premium of individual stocks. While the roles have been studied separately, the interactive relation in explaining the time variation of individual stock returns has been commonly ignored. This paper shows that they have a reinforcing effect. The price matters more for stocks with a high negative quantity, and quantity matters more for stocks with a high negative price.

The following sections provide an empirical test of this model. That there is an interactive role between the price and the quantity of variance risk in explaining the stock risk premium. The role of market variance risk is studied, and various implications are tested.

III. Data and Estimation

The main sample of this paper consists of stocks from NYSE, AMEX, and NASDAQ that have options traded actively in the market between 1996 and 2016. The option-implied volatility is obtained from OptionMetrics, intraday trading data from Trade and Quote, and individual stock data from the Center for Research in Security Prices (CRSP). Factor returns are from Ken French data library, and data on selected macroeconomic variables are from several different sources and summarized in Appendix A.2.

Among individual stock options, those that expire in the following month are selected. Several other filters are also applied to the data to minimize possible data errors. First, following previous studies such as Goyal and Saretto (2009), options in which the ask price is lower than the bid price, options whose bid price is equal to zero, and options whose price violates arbitrage bounds are eliminated. Second, options with zero open interest are also deleted. Third, options that have moneyness smaller than 0.95 or higher than 1.05 are removed. Among these options, one call and one put option closest to at the money (ATM) are chosen, and the simple averages of the Black-Scholes option-implied volatility between the call and the put are computed. The implied variance (IV) of an individual stock is the square of this average. When IV is missing, the value remains unchanged from the previous day. The option-implied variances are then matched to CRSP using both CUSIP and tickers.

The price of variance risk is defined as the covariance of the variance risk and the pricing kernel. Carr and Wu (2009) show that the price of variance risk can be approximately estimated by the difference between the variance under the physical measure and the risk-neutral measure. The equation follows from[8]

$$Cov_t(-SDF_{t+1}, \sigma^2_{i,t+1}) \approx E_t[\sigma^2_{i,t+1}] - E^Q_t[\sigma^2_{i,t+1}], \tag{9}$$

where SDF_t is the stochastic discount factor and $\sigma^2_{i,t}$ is the variance of stock i at time t.

Following this relationship, the price of variance risk of stock i ($\lambda_{v,i,t}$) is estimated in several alternative manners. Measuring the risk-neutral component of variance is relatively straight-forward if stocks have options traded. I use the Black-Scholes implied variance (IV) of at-the-money options as a proxy for the risk-neutral expectation. In defining the price of variance risk, I choose the estimate of the real-world counterpart to be as closely matched to

[8]The approximation is due to the risk-free rate. The relationship follows from the fact that $E^Q[\sigma^2_{i,t+1}] = E[SDF_{t+1} \cdot \sigma^2_{i,t+1}]$.

that of the risk-neutral component. In this paper, the real-world counterpart in two different ways.

I use three different measures for the price of risk. First, I use intra-day trading data obtained from Trade and Quote (1996-2016) to estimate the monthly historical realized variance. The first measure follows Han and Zhou (2012) and is constructed by using the end-of-month value of the option-implied variance and the RV estimated as the sum of 75-minute squared returns over the past month. For the RV, I use the average over five subsamples (Hansen and Lunde 2006) to deal with possible microstructure noise. The 75-minute intra-day interval for the individual stock return RV estimate follows earlier works of Bollerslev, Li, and Todorov (2016). The sampling interval is longer than what is typically used for the market index but reflects a higher likelihood of microstructure noise at the individual stock level. However, the main results are unaffected by choice of the sampling interval. Furthermore, while RV estimated in this manner may be inaccurate on a daily basis, they are only used after summing up to the monthly level. I also exclude illiquid stocks from the sample by removing all observations when stock prices do not move for ten consecutive days.

Although the first is simple to measure, it can also potentially be biased. First, the implied volatility of Black and Scholes (1973) is computed using pairs of ATM call and put options. This leads to a downward bias, particularly if the jump risk premium constitutes a substantial proportion of the variance risk premium.[9] To deal with the mismatch in these two components, alternatively, I replace RV with the realized bi-power variation (BPV) to estimate the price of variance risk. Barndorff-Nielsen and Shephard (2004) show that BPV is relatively robust to rare jumps and provides a model-free and consistent alternative to RV. BPV is estimated as

$$BPV_{i,t} = \sum_{k=1}^{K-1} |R_{i,t,k}||R_{i,t,k+1}| \tag{10}$$

[9]Bakshi, Kapadia, and Madan (2003) show that the risk-neutral expectation of the quadratic variation can be estimated using out-of-the-money (OTM) options. Previous studies – e.g., Bates (2000), Pan (2002), Bollerslev and Todorov (2011), Bollerslev, Todorov, and Xu (2015) – also show that deep OTM puts contain information about the negative jump risk premium. See also Carr and Madan (1999) and Andersen, Bondarenko, and Gonzalez-Perez (2015) on corridor-implied volatility, which indicates that excluding deep OTM options generates a downward bias in the risk-neutral expectation of quadratic variations.

where K is the total number of 75-minute intervals in the month. This measure is also averaged over five subsamples. Then, the price of variance risk using the BPV is

$$\lambda_{i,v,t}^{BPV} = BPV_{i,t} - IV_{i,t} \tag{11}$$

Two other problems remain with these two measures. First, RV (BPV) must be persistent and perform well in predicting future variances, i.e., forward-looking. Second, the two components should be estimated simultaneously. For example, using the monthly average for the first component while using end-of-month values for the second may be biased if there is a variance trend that is somehow correlated with how risk is priced.

A standard alternative for the market index is to use a variance forecast model-based measure for the real-world component. I follow this alternative and estimate the stock level variance using a standard generalized autoregressive conditional heteroscedasticity model (GARCH) of order (1,1). Beyond the fact that the variance is forecast based and that it allows having a slightly longer sample (1996-2017), a second benefit is that one can apply the model for relatively illiquid stocks even when intra-day trading is not too frequent. That the stock-level variance can be estimated for all stocks is later used to show that the key result of this paper is extendable even for stocks that do not have options traded.

The price of variance risk where the physical measure is estimated using a GARCH (1,1) model is defined as

$$\lambda_{v,i,t}^{G} = \hat{\sigma}_{i,t}^{2} - IV_{i,t}, \tag{12}$$

where $\hat{\sigma}_t^2$ is the GARCH estimate evaluated at the end of each month, estimated on a rolling basis using three years of data.

Multiple specifications are used to estimate the quantity of stock variance risk. As the main specification, similar to Ang, Hodrick, Xing, and Zhang (2006), the variance risk exposure is the slope of the two-variable regression

$$R_{i,t} = \beta_{0,i} + \beta_{m,i}\ R_{m,t} + \beta_{v,i}\ [IV_{i,t} - IV_{i,t-1}] + \epsilon_{i,t}, \tag{13}$$

where R_m is the excess market returns, and R_i is the excess stock returns of stock i. The slope, $\hat{\beta}_{v,i}$, is estimated every month using past six months of data. As an alternative, I also consider a single-variable regression by excluding the market factor. The key results of this paper holds regardless of the exact specification used to estimate the slope.

One alternative way to obtain the quantity is to directly use the variance estimated from the GARCH (1,1) model. The GARCH-based quantity of risk ($\beta^{G}_{v,i}$) is obtained by replacing the IVs in the above regression with the as the slope of the regression GARCH-based estimate of stock variance $\hat{\sigma}^2_{i,t}$.[10] Unlike the option-based estimate, the GARCH-based quantity estimate can be estimated for all stocks, even when stocks do not have options traded.

Table I summarizes the mean, median, and standard deviations of the sample considered in this paper. There is a total of 245,248 stock-month during the sample period. While the entire CRSP sample has an average of 4,948 stocks in a given month during the period of 1996/01-2015/12, on average, 977 stocks have options traded actively in the market. There are 231 stocks at the beginning of 1996 and 1,775 stocks at the end of the sample. The median of the IV is 0.011 (36.3% annualized), while the RV has a median of 0.007 (29.0% annualized). The BPV has a smaller median of 0.002 (15.4% annualized). The mean of IV is 0.018, the mean of RV is 0.014, and the mean of BPV is 0.003, which suggest that the cross-sectional distribution of the variance is slightly positively skewed. Outliers in stock variance is not necessarily a severe concern in the context of this paper, as they are not influential when forming portfolios based on their rankings.

[10]In some subsequent analysis, I choose to estimate this quantity using a relatively shorter estimation window of three months. As will be described later, some of the cross-sectional tests use on the quantity of variance risk as a dependent variable, which may lead the standard errors to be biased mechanically due to using overlapping observations.

The table also suggests that the price of variance is mostly negative; 72.5% of the stocks in the sample have a negative price of risk if RV is used. The difference between the RV and IV is negative on average, but a sizable proportion remains positive, which is consistent with other previous research (e.g., Goyal and Saretto). Although a substantial portion of stocks may have a positively estimated price of risk, partly arising from estimation error in RV, some downward bias in IV may also exist by using only ATM options in the estimation. Notably, if BPV is used instead, only 2% of the stocks on average have a positively estimated price of variance risk.

The variance exposures are negative, on average, suggesting that for the average stock, high uncertainty is regarded to be unfavorable for the investor. However, for a notable number of stocks (20.8%), returns are positively related to contemporaneous variance movements, which is partly due to growth or real options. Compared with the entire CRSP database, the chosen sample has a similar average market beta. Also, stocks in the sample tend to be larger than the CRSP average. The tendency is natural as the sample systematically excludes options on these stocks that are less likely to be traded.

IV. Empirical Results

The framework of this paper suggests an interactive relationship between the price and quantity of stock variance risk for the individual stock risk premium. Empirically, we expect the quantity of stock variance risk to affect stock returns more when the price of risk is highly negative. Similarly, the price should matter more for stocks with a high negative quantity. This section provides empirical evidence that supports the framework, suggesting that both price and quantity are important in explaining the time variation of individual stock returns.

1. Main Results: Interactive Role of the Price and Quantity

Previous studies find that either the price or quantity of stock variance risk, separately, is priced among individual stocks. Using a similar measure to this paper, Bali and Hovakimian (2009) and Han and Zhou (2012) show that stocks that have a more negative price of variance risk tend to have higher future returns. These studies use a similar measure to that of this paper but have different interpretations. For example, Han and Zhou (2012) argue that the difference proxies for aggregate variance risk exposures and report a negative relationship between the price of variance risk and subsequent returns. Bali and Hovakimian (2009) regard this spread as a proxy for the jump risk of the stock.

While academic literature does not focus much on the quantity of stock variance risk itself, it is well-known that the quantity of market variance risk is priced.[11] They are intimately connected because the market portfolio is an aggregate of individual stocks. While the market variance is also likely to be affected by the correlation structure of individual stock returns, higher market variance generally increases the stock variance.

I first analyze how the two measures are related to stock performances by forming single-sorted portfolios. I first estimate the price and the quantity of variance risk for each stock. Then, the stocks are sorted either by the price or the quantity of variance risk. The stocks are divided into quintiles based on the estimates, and the simple averages of the price and quantity of variance risk estimates, value-weighted returns, risk-adjusted returns, market beta, and the market capitalization are computed and reported. Risk-adjusted returns, denoted by α_6, are returns controlled for the size, value, profitability, investment (Fama and French 2015), and momentum (Carhart 1997). Both returns and risk-adjusted returns are evaluated over the subsequent month after the formation, and the units in tables are in monthly returns.

Table II summarizes the performance of the single-sorted portfolios. Panel A summarizes the returns of the price-sorted portfolios. The first two columns of Panel A confirm earlier

[11]See, for example, Coval and Shumway (2001), Ang, Hodrick, Xing, and Zhang (2006), Adrian and Rosenberg (2008), Bali and Hovakimian (2009), Cremers, Halling, and Weinbaum (2015) and Chang, Christoffersen, and Jacobs (2013).

studies that find stocks with a high negative price of variance risk have higher subsequent returns. The difference between the top and bottom quintiles is 0.75% in excess returns and is statistically significant. The next two columns summarize the averages of the contemporaneous estimates of the price and quantity of variance risk. On average, stocks with a high negative price of variance risk also have a smaller variance risk exposure. Lastly, the last few columns provide some additional summary statistics. Stocks with a more negative price of variance risk are smaller in size, have higher market betas, and have relatively positive exposure to variance risk.

Panel B shows the performances of the quantity-sorted portfolios. The first two columns summarize the excess returns and risk-adjusted returns. I find limited evidence that the quantity of variance risk is priced. While stocks with a high negative risk exposure slightly outperform stocks with a small or positive variance risk exposure, the difference is tiny and statistically insignificant. Although the quantity of stock variance risk is different from the quantity of market variance risk, this result is similar to the result of Chang, Christoffersen, and Jacobs (2013) that suggests a weak relationship with the market variance risk exposure if more than a single month of data is used for estimation.

It is also worth noting that the quantity sorted portfolios show no difference in the price of risk between the top and bottom quintiles. This result suggests that the price and the quantity contain some different set of information about stock characteristics.

To understand the role of growth options and the liability constraint, I also compute the averages of several other firm-specific variables of interest for each quintile. The appendix describes the exact definitions and calculations of these variables. The mean of these variables for the portfolios is summarized in subsequent columns of Panel B. On average, stocks with positive exposure to their variance shocks are smaller in size and have higher leverage, more growth options, and lower profitability. These results suggest that the quantity is related to firm-specific variables that are connected to the limited liability constraint or growth options. The results of this table should be interpreted as only indicative since the t-statistics for the difference between the top and bottom quintiles are not calculated, due to the highly serially correlated nature of the firm-specific variables as well as the estimates.

The primary focus of this paper is on whether an interactive or reinforcing relationship exists between the price and quantity of variance risk. If so, the performance of the long-short portfolios of a single sort will depend on the level of the other sort. In particular, the long-short portfolio returns of the risk exposure should be much stronger and more prominent when the price of risk is highly negative. The long-short portfolio returns concerning the price of risk may be significant mainly for stocks whose variance risk exposure is highly negative. This possibility is evaluated by analyzing the performance of the price and the quantity double-sorted portfolios.

Table III summarizes the performance of the double-sorted portfolios, where the price and quantity of variance risk are estimated using the basic specification, as discussed in the previous section. All stocks are sorted independently by the price and the quantity of variance risk and divided into four groups. As a result, a total of 16 portfolios is formed. The time-series average of the value-weighted returns and the six-factor risk-adjusted returns of the following month are evaluated. The t-statistics are adjusted for heteroskedasticity and autocorrelation following the Newey and West (1987) methodology. Each row represents portfolios with different price levels, and each column shows those with varying levels of quantity. The price of variance risk is estimated in several ways, as described in the previous section.

Panel A shows the result when the RV is used to estimate the price of variance risk. As predicted by the model, the price of variance risk has the most substantial impact on future stock returns if they are highly negatively exposed to their variance risk. For stocks that have a high negative quantity of risk, the spread between the high minus low price-sorted portfolios is -1.35% in monthly excess returns and -1.55% in risk-adjusted returns, but reduces to -0.10% in excess returns and -0.46% in risk-adjusted returns, for stocks with a relatively positive quantity of risk. Analogously, when the price of variance risk is highly negative, the spread between high and low quantity-sorted portfolios is -0.91% in excess returns and -1.00%

in risk-adjusted returns. The spread has the opposite sign with 0.34% in excess returns and 0.09% in risk-adjusted returns for stocks with a relatively positive price of risk.

Most importantly, the difference between these two spreads is highly statistically significant at 1% for both excess returns (1.25%) and risk-adjusted returns (1.09%). In conclusion, the variance risk exposure mostly affects stock returns, whose price of variance risk is highly negative. The price of risk matters most for stocks with a negative exposure to variance risk. In other words, empirically, a strong and statistically significant interactive effect is observed.

As noted, the IV of stocks may not represent the risk-neutral expectation of the RV equivalent if, for example, jump risk is priced. If this is the case, BPV may be better paired with IV. Panel B summarizes an equivalent result of the double-sorted portfolios when BPV is used instead of RV. Overall, the results are very similar. The difference in spread decreases to 1.07% in excess returns but increases to 1.61% in risk-adjusted returns. They are both highly statistically significant.

Finally, Panel C shows the results when the price and the quantity are estimated based on the parametric GARCH(1,1) model. As described, this specification has several benefits. First, the two components of the price of risk are estimated simultaneously and equivalently on a forward-looking measure. Second, the theory suggests that the beta should measure the exposure to the underlying risk, which are the variance innovations under real-world measures. Hence, the quantity estimation is also more in line with the theory. The results of this table suggest that the difference in the spread remains highly statistically and economically significant for both excess returns and risk-adjusted returns. The t-statistics are slightly higher than those of the main analysis, likely because the underlying risk is estimated from real-world measures.

Table IV considers several alternative specifications of the RV-based result. First, I consider different sorting methodologies. If the price and quantity are highly correlated in the cross-section, implementing different ways of sorting may affect the result. Panel A shows the results of the dependent sort when stocks are first sorted by the price, whereas Panel B is when the quantity is the first sorting variable. The results are largely consistent with those of the independent sort. The difference in the performance of the one-way sort decreases somewhat to 0.80%, reflecting the fact that the price sorted portfolio takes some of the variations in the quantity of variance risk. However, the difference in performance remains statistically significant.

Second, the quantity is estimated alternatively using weighted least squares (WLS). WLS is useful as they are more efficient than the ordinary least squares when there is heteroscedasticity. The results of Panel C suggest that this is indeed the case, as they show slightly stronger performance. The higher spread for the WLS-based sort may suggest that higher variance-of-variance of stock returns are associated with high variance of returns.

In Panel D, the quantity ($\beta^1_{v,i}$) is estimated using a single factor regression, excluding the market factor.

$$R_{i,t} = \beta_{0,i} + \beta^1_{v,i}[IV_{i,t} - IV_{i,t-1}] + \epsilon_{i,t}. \tag{14}$$

If market variance risk is indeed priced among individual stocks, we expect to see even stronger results for this alternative specification. The result in Panel D confirms that market variance may constitute a substantial part of variance that is priced among individual stocks. The difference in the performances increases to 1.82% in excess returns and 1.83% in risk-adjusted returns. They are highly statistically significant.

2. The Market Variance Risk Exposure

How stock prices respond to changes in market variance and to changes to their variance are closely connected. If stock variances move in the same direction as those of the market, it would generate a high correlation between the quantity of variance risk and the market variance beta.

In two somewhat independent settings, I provide evidence that suggests the reported interactive relationship is not likely to be driven by the stocks' difference in their market variance risk exposures. The first follows from further decomposing both the price and quantity of variance risk into two components – one that represents those driven by market variance risk and the other that represents those from variance shocks orthogonal to the market variance. If the interactive relationship is mostly due to the differences in the market variance risk exposures, these effects should disappear if stocks are sorted by those of the idiosyncratic component. The second is from the presumption that a market variance factor should capture the exposures to market variance risk. Hence, risk-adjusting returns should not absorb the interactive relationship if it is not due to market variance.

The break-down of market and idiosyncratic variance risk and the calculation of the price and quantity of variance risk is implemented by first assuming a market model for excess returns of stock i ($R_{i,t}$).

$$R_{i,t} = a_i + \beta_{m,i} R_{m,t} + \epsilon_{i,t},$$

The sum of the intercept and the residuals ($a_i + \epsilon_{i,t}$) captures the stock price variations related to changes in systematic risk factors other than the market factor and also, possibly, any idiosyncratic shock. The above specification also implies that the stock variance process follows a single factor structure.

$$\Delta Var_t(R_{i,t+1}) = \beta_{m,i}^2 \Delta Var_t(R_{m,t+1}) + \Delta Var_t(\epsilon_{i,t}). \tag{15}$$

From the above equation, the idiosyncratic variance shock ($\Delta Var_t(\epsilon_{i,t})$) is orthogonal to market variance shocks. Hence, the variance risk of a stock i can be represented as the sum of the component related to changes in market variance and another related to non-market variance shocks.

Empirically, the slopes and residuals of Equation (15) are estimated by regressing changes in option-implied variance of stock i ($\Delta IV_{i,t}$) on the equivalent change of variance of the market index. I use the VIX index as a proxy for market option-implied variance. I estimate

$$\Delta IV_{i,t} = \beta_{vx,0,i} + \beta_{vx,i} \Delta VIX_t^2 + e_{i,t} \tag{16}$$

using six months of daily data. The series of residuals ($\hat{e}_{i,t}$) is the idiosyncratic variance risk estimated from the market model. Then, a stock's the exposure to this residual ($\beta_{e,i}$), can be estimated from the slope of a regression of stock returns on the residuals.

$$R_{i,t} = \beta_{0,i} + \beta_{m,i}\ R_{m,t} + \beta_{e,i}\ \hat{e}_{i,t} + \epsilon_{i,t} \tag{17}$$

That is, the slope of Equation (17) measures how a stock return reacts to changes in variance orthogonal to market variance shocks, which is referred to as *the quantity of idiosyncratic variance risk*.

Also, using Equation (16, the price of variance risk can be decomposed into

$$\begin{aligned}\mathrm{Cov}_t(-SDF_{t+1}, \Delta Var_t(R_{i,t+1})) = \beta_{vx,i}\mathrm{Cov}_t(-SDF_{t+1}, \Delta\mathrm{Var}_t(R_{m,t+1}))\\ + \mathrm{Cov}_t(-SDF_{t+1}, e_{i,t+1}).\end{aligned}$$

From this relationship, the price of variance risk due to the idiosyncratic component can be rewritten as,

$$\mathrm{Cov}_t(-SDF_{t+1}, \Delta e_{i,t+1}) = \mathrm{Cov}_t(-SDF_{t+1}, \Delta Var_t(R_{i,t+1})) - \hat{\beta}_{vx,i}(RV_{m,t} - VIX_t^2). \tag{18}$$

The above equation is equivalent to

$$\lambda_{e,i,t} = \lambda_{v,i,t} - \hat{\beta}_{vx,i,t}\lambda_{x,t}, \tag{19}$$

where $\lambda_{x,t}$ is the price of market variance risk. The first component of the right-hand side can be estimated as the difference between the RV and the IV of the stock. The left-hand side, $\lambda_{e,i,t}$, is the *price of idiosyncratic variance risk* of the market model and is computed by subtracting the scaled price of market variance risk from the price of variance risk of individual stock returns.

Panel A of Table V shows the performance of the portfolios sorted by the price and the quantity of idiosyncratic variance risk. The difference in spreads for the double-sorted portfolios is 1.13% per month in excess returns and 1.07% in risk-adjusted returns. These numbers are slightly lower than those of Table III but is not too much different. This result together with the last panel of Table IV suggests that while a small proportion of the price variance risk of individual stock returns may be related to market variance risk, a substantial part remains uncaptured.

Alternatively, it is possible to adjust the returns using a traded market variance risk factor. I follow Ang, Hodrick, Xing, and Zhang (2006) to construct the factor-mimicking portfolio (FVIX) of the market variance risk. Panel B of Table V provides the seven-factor alphas of the double-sorted portfolios. The returns are adjusted for the market, size, value, momentum, profitability, investment, and FVIX factors. The results are similar to those of Panel A. The spread in the alphas is 1.16%, which is comparable to that without adjusting for the FVIX factor.

In short, there is limited evidence that suggests market variance risk plays a vital role in explaining the interactive relationship between the price and quantity of variance risk. Furthermore, the result of this section indicates that some systematic risk factors orthogonal to the market may also be priced and determine the short-term variation of individual stock returns.

V. Model Implications

The main analysis confirms that variance risk, beyond that of the market factor, is an important source of risk that is priced among individual stocks. Moreover, the price and the quantity of variance risk are two dimensions that represent a fraction of the stock risk premium.

In this section, we first test the two implications of the framework. One that the quantity should be determined by the factor betas and the factor return-variance correlation, and two that price of variance risk should be higher for stocks that have high absolute betas. Then, by extrapolating the risk-neutral expectation of variance from the latter relationship, I extend the sample to include all stocks that do not have options traded. I discuss the key implications of this extension. Finally, I conclude the section by performing a cross-sectional asset pricing test that includes both the price and the quantity of variance risk.

1. The Quantity of Variance Risk, Factor Betas, and Factor Return-Variance Covariance

The framework of this paper suggests that the quantity of variance risk depends on a stock's factor exposures and the factors' return-variance covariance. The relationship between factor betas, the variance beta of the factor, and the quantity of the stock variance risk are tested in a limited setting with Fama-French factors. I limit the analysis to traded factors because it is relatively easier to estimate the variance of these factors compared to macroeconomic factors, only observed at the monthly or quarterly frequency.

I first estimate the variance of each of the factors using a GARCH(1,1) model, in a similar manner applied to individual stocks. The variance beta of a traded factor (F_j) is then estimated by regressing daily factor returns on the innovations in the factor variance forecasts. I estimate the beta ($l_{1,j}$) of the regression of

$$F_{j,t} = l_{0,j} + l_{1,j}(V_{F,j,t} - V_{F,j,t-1}) + \eta_{j,t}, \tag{20}$$

where $V_{F,j}$ is the variance forecast of factor j. These regressions are estimated from a rolling window of three months.

The goal is to study how the quantity of variance risk is related to the joint combination of the stock's factor betas and the factors' variance betas ($l_{1,j}$). To take advantage of the larger sample of that includes stocks both with and without options, the GARCH-based quantity of variance risk is used as the dependent variable. I test whether the quantity is positively or negatively related to the factor betas of stocks depending on the sign of the variance betas. I implement a two-stage analysis. First, I estimate the cross-sectional regression of

$$\hat{\beta}^{G}_{v,i,t} = \delta_{0,t} + \sum_{j} \delta_{j,t}\hat{\beta}_{j,i,t} + \epsilon_{i,t}$$

every quarter, where $\hat{\beta}_{j,i,t}$ is the factor jth beta of stock i. To avoid the standard errors being affected mechanically by using overlapping observations, all betas in this regression are estimated using a three-month rolling window.

The framework suggests that $\delta_{j,t}$ should have the same sign as the variance beta of the factors. If this is true, we expect a positive relationship between $\hat{\delta}_{j,t}$ and $\hat{l}_{j,t}$. Therefore, as a second step, the relationship is tested on a factor-by-factor basis by estimating the time-series regression of

$$\hat{\delta}_{j,t} = \gamma_{0,j} + \gamma_j \hat{l}_{j,t} + e_{j,t}.$$

Whether any signs of γ_j are positive or negative is of interest. Table VI summarizes the results of the time-series regressions. Panel A shows the results when both market factors and stock-level variance shocks are included to estimate the quantity of variance risk. Among the six factors considered, two factors – value and momentum – are consistent with the explanation. The signs of two additional factors – investment and profitability factors – are consistent with the hypothesis, but they are statistically insignificant.

Notably, the market factor has a negative slope coefficient, which at first sight may look contradictory to the framework. However, this is partly related to how the quantity of variance risk is estimated. The quantity is estimated with the market factor as an additional variable, which partly takes the variation of stock returns related to market variance shocks. To see whether the opposite sign observed may be related to control of the market factor, in Panel B, I estimate the quantity of variance risk from a single factor regression as in Equation (14) using the variance forecasts of the GARCH (1,1). Although the slopes are statistically insignificant, as conjectured, the coefficient for the market beta is no longer negative.

In conclusion, the analysis of Table VI suggests that the stock betas and the factors' return-variance correlation jointly determine the quantity of the stock variance risk. When factor variance betas are negative, the quantity of variance risk and factor betas are likely negatively related. When the variance betas are positive, they are likely to be positively related.

2. Determinants of the Price of Variance Risk

The framework of this paper suggests that variance is positively related to the absolute size of the factor betas. If we assume a linear factor structure, the stock-level variances should be linear in the squared of the factor betas. This relationship can be observed if we take the variance function of an arbitrary linear factor model. If stock i's return (R_i) is represented as a linear combination of the sum of n independent factors ($F_j, j = 1 \ldots, n$) as

$$R_{i,t} = \beta_{0,i} + \sum_{j=1}^{n} \beta_{j,i} F_{j,t} + \epsilon_{i,t}, \tag{21}$$

the variance process of stock i can be represented as

$$Var(R_{i,t}) = \sum_{j=1}^{n} \beta_{j,i}^2 Var(F_{j,t}) + Var(\epsilon_{i,t}). \tag{22}$$

Then, assuming the idiosyncratic variance is not priced, the stock-specific price of variance risk can be represented as

$$\lambda_{v,i,t} = \sum_{j=1}^{n} \beta_{j,i}^2 \lambda_{v,j,t}, \tag{23}$$

where $\lambda_{v,j,t}$ is the price of jth factor variance risk.

I test the relationship between the price of variance risk and squared betas by first sorting stocks by their corresponding factor betas and forming decile portfolios. Then, for each portfolio, I evaluate the equally-weighted average of the price of variance risk. Figure 1 shows the relationship. Each figure represent a different factor, the solid line represents the average value of the betas, and the dashed line represents the average value of the price of variance risk.

Two observations are worth noting from these figures. First, the price of variance risk shows a hump-shaped pattern for most factors, excluding the market, where most stocks have a positive beta. Second, the point where the solid line meets the horizontal line is where the betas are close to zero. For the portfolios whose betas are close to zero, the price of variance risk reaches its maximum – a value that is also close to zero.

To formally test the hypothesis, I first estimate the cross-sectional regression of (22) and (23) using option-implied variance as dependent variable and stock betas as independent variables. I repeat this by replacing variances with the price of risk. Then, I evaluate the time-series average of the slopes along with the Newey-West adjusted t-statistics. The first stage regression is estimated only every six months reflecting the fact that the betas are estimated using a six-month regression.[12]

Table VII show the results of the regressions. The dependent variable in Panel A and B is the option-implied variance, and for Panel C and D, it is the price of variance risk. The independent variables in Panel A and C are the betas of the macroeconomic factors as defined in Appendix A.2, whereas for Panel B and D, it is the Fama-French six-factor betas.

Focusing on the first two panels, I find a robust positive relationship between the beta squares and option-implied variances. Stocks with high squared betas, regardless of whether it is a macroeconomic or a Fama-French type of factor, have higher variance. The t-statistics are mostly above two, with some reaching the value of 8. These panels suggest that stocks that have high betas, regardless of their signs, tend to be more volatile.

A similar relationship holds for the price of variance risk, with the opposite sign. From (23), the price of the stock variance risk should be more negative (positive) for stocks with a

[12]The choice of this interval is to be conservative and to avoid the possibility that the standard errors are biased through mechanical reasons. The t-statistics are much higher as the frequency of evaluation is reduced. For example, when the regressions are estimated monthly, the estimates are all statistically significant with the t-statistics of > 10. An alternative is to estimate the quantity using a three-month regression and evaluate the relationship quarterly. This alternative methodology also generates a higher t-statistics.

high squared beta if the price of the factor variance risk is negative (positive). The last two panels confirm that it is more negative when the beta squared is higher. The coefficients are mostly statistically significant with a large t-statistics with a few exceptions.

First, the market factor is insignificant if both size and value factors are added in the regression. However, as Figure 1 also shows, an unreported analysis shows that the relationship is negative and statistically significant when the market factor is included as a sole independent variable. Also, the squared beta of the market factor is statistically significant if we add macro-factors as if they were state variables that explain the economic conditions in addition to the market. Second, consumption growth and production growth are statistically significant if considered separately but become insignificant if added together. This observation suggests that consumption and production volatility betas are highly correlated across stocks.

In conclusion, the results of Table VII suggests that the variances are higher if stocks have either a highly positive or negative exposure to a risk factor, and the price of variance risk is more negative for these stocks. In the following section, I use the observation of this table to construct a predicted risk-neutral expectation of variance that can be used to evaluate the relationship for all stocks.

3. Informed Trading - Extension to All Stocks

One may think that the main result of this paper, the interactive role of the price and the quantity, is related to informed trading. Privately informed investors may simultaneously affect the prices of options and stocks by trading on the information. For example, negative news to a firm may generate a positive price pressure on put options and a negative one for stocks.[13] These stocks would have a negative quantity of risk and relative positive price of variance risk because option-implied volatility will be higher. According to the framework,

[13] See, for example, Cremers and Weinbaum (2010), Xing, Zhang, and Zhao (2010), An, Ang, Bali, and Cakici (2014), among others.

these stocks should have a lower risk premium. Hence, the framework is not able to completely distinguish whether the low stock returns are due to price pressure or risk premium.

In contrasts, investors trading on positive information is not consistent with the result. Positive news to a firm may both push option and stock prices up, leading to a positive quantity of risk. Since option-implied variance will be higher, the price of risk is also small. Hence, according to the framework, the risk premium is low, which is inconsistent with informed trading affecting stock returns.

The possibility that informed trading channel may be driving the main result is tested by showing that the interactive patterns of the price and quantity, as a determinant of future stock returns, is present even for stocks that do not have options traded. If the price pressure of informed traders is deriving the result, the interactive role shown should disappear for stocks that do not have options traded. To extend the analysis for the entire sample, I use the strong relationship between squared factor betas and the option-implied variances reported earlier to indirectly measure the risk-neutral variance of all stocks.

Beyond testing the informed trading channel, this extension has two other purposes. First, applying the relationship to non-optionable stocks serves as an out-of-sample test. It establishes the robustness of the result. Second, higher statistical power can be obtained with more stocks, and asset pricing tests are implementable that would otherwise not be possible with a limited number of stocks.

The estimation of the risk-neutral expectation is implemented from the cross-sectional relationship that variances should be higher for stocks with high absolute betas. Every month, a cross-sectional regression of (22) is estimated using both the macroeconomic and Fama-French factors. Then, the predicted value of the option-implied variances can be computed for both optionable and non-optionable stocks. The predicted price of variance risk ($\widehat{\lambda}^P_{v,i,t}$) is the difference between the GARCH(1,1)-based variance forecast and the predicted option-implied variance.

Table VIII summarizes the performances of the double-sorted portfolios formed on the predicted price of variance risk. Panel A and B summarize the results when both stocks with and without options traded are included in the sample. Panel A is when macro factors are used, and Panel B is when Fama-French factors are used to estimate the risk-neutral expectation. Generally, the results are slightly weaker than those of the main analysis. While the difference in the spread is statistically significant, with 0.86-0.87% for the macro-factor based analysis, 1.15-1.14% for the Fama-French factor-based analysis, some of the monotonicity is violated. However, the patterns are consistent with the main analysis that high negative quantity of variance risk stocks outperform only when the price of variance risk is also highly negative.

The strength of the relationship further decreases when optionable stocks are entirely removed from the analysis. Thus, the sample is left with stocks that are small and less liquid. Panel C and D show the results when stocks with options traded are entirely removed from the sample. Panel C is when macro factors are used, and Panel D is when Fama-French factors are used to estimate the risk-neutral expectation. The difference in the long-short portfolios formed based on a single dimension across different groups of the second dimension decreases to 0.68% – 0.94% and is only marginally statistically significant. However, even for this limited sample, similar patterns are observed.

4. Cross-sectional Asset Pricing Test

The stock-specific price of variance risk is related to the factor structure of stock returns. The quantity is also related to the factor betas of stock as well as the signs of the return-variance covariance of the factors. As both the price and the quantity are connected to the factor structure, one may think that they should contain similar information. To confirm that they contain separate information, I implement a cross-sectional test of stock returns using the predicted price and quantity simultaneously.

To utilize information on the entire stock universe, I obtain the predicted price of variance risk and the quantity of variance risk from GARCH (1,1). Then, for each month t, I estimate the regression of

$$R_{i,t+1} = b_0 + b_1 \hat{\lambda}^P_{v,i,t} + b_2 \hat{\beta}^G_{v,i,t} + \mathbf{c}'\mathbf{Control}_{i,t} + \epsilon_{t+1},$$

where $\mathbf{Control}_{i,t}$ is a vector of control variables, which can be a combination of the beta of a factor (e.g., the market beta) or some firm characteristics. I consider size, the log of book-to-market ratio (B/M), momentum (returns between $t-1$ to $t-12$), the CIV beta (common factors of idiosyncratic volatility of Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016)), idiosyncratic volatility (Ivol), the growth rate in asset (Growth), and profitability measured as the return-on-equity. Finally, following Fama and MacBeth (1973), I take the time-series average of the coefficient and report the Newey-West adjusted t-statistics.

Table IX provides the results of this cross-sectional regression. Panel A shows the result when the risk-neutral expectation of variance is estimated from macro-factor betas, and Panel B is when it is estimated from Fama-French factor betas. The coefficients are scaled so that they have a value between 0.01 and 10. Overall, the cross-sectional asset pricing test confirms the results of the main analysis. Future stock returns are negatively related to the price of variance risk after controlling for the quantity and are also negatively related to the quantity controlling for the price. They are significant after controlling for several variables, including the CIV beta, growth and profitability of the firm. This table suggests that both the price and quantity are important determinants of future stock returns that contain independent information about the stock risk premium.

VI. Conclusion

This paper investigates the interactive role of the price and quantity of variance risk. The findings suggest that these two dimensions are interrelated. When stocks are sorted by the quantity of variance risk of their returns, those stocks that are more exposed to their variance risk tend to have higher subsequent returns. However, this effect is substantially stronger for stocks whose price of variance risk is also highly negative. For stocks with a small or positive price, the risk exposure does not matter. Similarly, the price of variance risk matters most for stocks that have high negative exposure to variance risk. Conclusively, the price of individual stock variance risk and the quantity of variance risk have an interactive relationship.

This paper finds that the interactive relationships are mainly driven by the variance risk of factors other than the market factor. The interactive relationship exists even when market variance risk is controlled or when stocks are sorted by the price and quantity of non-market variance risk. The results can be extended to all traded stocks, which suggests that the results are unlikely to be driven by price pressures in one or the other form.

References

Adrian, Tobias, and Joshua Rosenberg, 2008, Stock Returns and Volatility: Pricing the Short-Run and Long-Run Components of Market Risk, *The Journal of Finance* 63, 2997–3030.

An, Byeong-Je, Andrew Ang, Turan G. Bali, and Nusret Cakici, 2014, The Joint Cross Section of Stocks and Options, *The Journal of Finance* 69, 2279–2337.

Andersen, Torben G., Oleg Bondarenko, and Maria T. Gonzalez-Perez, 2015, Exploring Return Dynamics via Corridor Implied Volatility, *Review of Financial Studies* 28, 2902–2945.

Ang, Andrew, Robert J Hodrick, Yuhang Xing, and Xiaoyan Zhang, 2006, The Cross-section of Volatility and Expected Returns, *The Journal of Finance* 61, 259–299.

Bakshi, Gurdip, Nikunj Kapadia, and Dilip Madan, 2003, Stock Return Characteristics, Skew Laws, and the Differential Pricing of Individual Equity Options, *The Review of Financial Studies* 16, 101–143.

Bali, Turan G., Stephen J. Brown, and Yi Tang, 2017, Is economic uncertainty priced in the cross-section of stock returns?, *Journal of Financial Economics* 126, 471–489.

Bali, Turan G, and Armen Hovakimian, 2009, Volatility Spreads and Expected Stock Returns, *Management Science* 55, 1797–1812.

Bansal, Ravi, Dana Kiku, Ivan Shaliastovich, and Amir Yaron, 2014, Volatility, the Macroeconomy, and Asset Prices, *The Journal of Finance* 69, 2471–2511.

Barinov, Alexander, 2013, Idiosyncratic Volatility, Growth Options, and the Cross-Section of Returns, *Working Paper*.

Barndorff-Nielsen, Ole E.;, and Neil Shephard, 2004, Power and Bipower Variation with Stochastic Volatility and Jumps, *Journal of Financial Econometrics* 2, 1–37.

Bates, David S, 2000, Post-'87 crash fears in the S&P 500 futures option market, *Journal of Econometrics* 94, 181–238.

Bekaert, Geert, Eric Engstrom, and Yuhang Xing, 2009, Risk, uncertainty, and asset prices, *Journal of Financial Economics* 91, 59–82.

Bekaert, Geert, and Marie Hoerova, 2014, The VIX, the Variance Premium and Stock Market Volatility, *Journal of Econometrics* 183, 181–192.

Bekaert, Geert, Marie Hoerova, and Marco Lo Duca, 2013, Risk, uncertainty and monetary policy, *Journal of Monetary Economics* 60, 771–788.

Black, Fischer, and Myron Scholes, 1973, The Pricing of Options and Corporate Liabilities, *Journal of Political Economy* 81, 637–654.

Boguth, Oliver, and Lars-Alexander Kuehn, 2013, Consumption Volatility Risk, *The Journal of Finance* 68, 2589–2615.

Bollerslev, Tim, Sophia Zhengzi Li, and Viktor Todorov, 2016, Roughing up beta: Continuous versus discontinuous betas and the cross section of expected stock returns, *Journal of Financial Economics* 120, 464–490.

Bollerslev, Tim, George Tauchen, and Hao Zhou, 2009, Expected Stock Returns and Variance Risk Premia, *Review of Financial Studies* 22, 4463–4492.

Bollerslev, Tim, and Viktor Todorov, 2011, Tails, Fears, and Risk Premia, *The Journal of Finance* 66, 2165–2211.

Bollerslev, Tim, Viktor Todorov, and Lai Xu, 2015, Tail risk premia and return predictability, *Journal of Financial Economics* 118, 113–134.

Cao, Charles, Timothy Simin, and Jing Zhao, 2008, Can Growth Options Explain the Trend in Idiosyncratic Risk?, *The Review of Financial Studies* 21, 2599–2633.

Carhart, Mark M, 1997, On Persistence in Mutual Fund Performance, *The Journal of Finance* 52, 57–82.

Carr, Peter, and Dilip Madan, 1999, Option valuation using the fast Fourier transform, *The Journal of Computational Finance* 2, 61–73.

Carr, Peter, and Liuren Wu, 2009, Variance Risk Premiums, *Review of Financial Studies* 22, 1311–1341.

Chang, Bo Young, Peter Christoffersen, and Kris Jacobs, 2013, Market Skewness Risk and the Cross Section of Stock Returns, *Journal of Financial Economics* 107, 46–68.

Chen, Nai-Fu, Richard Roll, and Stephen A Ross, 1986, Economic Forces and the Stock Market, *The Journal of Business* 59, 383–403.

Chen, Zhanhui, and Ralitsa Petkova, 2012, Does Idiosyncratic Volatility Proxy for Risk Exposure?, *The Review of Financial Studies* 25, 2745–2787.

Conrad, Jennifer, Robert F Dittmar, and Eric Ghysels, 2013, Ex Ante Skewness and Expected Stock Returns, *Journal of Finance* 68, 85–124.

Coval, Joshua D, and Tyler Shumway, 2001, Expected Option Returns, *The Journal of Finance* 56, 983–1009.

Cremers, Martjun, Michael Halling, and David Weinbaum, 2015, Aggregate Jump and Volatility Risk in the Cross-Section of Stock Returns, *The Journal of Finance* 70, 577–614.

Cremers, Martijn, and David Weinbaum, 2010, Deviations from put-call parity and stock return predictability, *Journal of Financial and Quantitative Analysis*.

David, Alexander, and Pietro Veronesi, 2014, Investors' and Central Bank's Uncertainty Embedded in Index Options, *Review of Financial Studies* 27, 1661–1716.

Drechsler, Itamar, and Amir Yaron, 2011, What's Vol Got to Do with It, *Review of Financial Studies* 24, 1–45.

Engle, Robert F., Eric Ghysels, and Bumjean Sohn, 2013, Stock Market Volatility and Macroeconomic Fundamentals, *Review of Economics and Statistics* 95, 776–797.

Fama, Eugene F., and Kenneth R. French, 2015, A five-factor asset pricing model, *Journal of Financial Economics* 116, 1–22.

Fama, Eugene F, and James D MacBeth, 1973, Risk, Return, and Equilibrium: Empirical Tests, *Journal of Political Economy* 81, 607–636.

French, Kenneth R, G William Schwert, and Robert F Stambaugh, 1987, Expected Stock Returns and Volatility, *Journal of Financial Economics* 19, 3–29.

Fu, Fangjian, 2009, Idiosyncratic risk and the cross-section of expected stock returns, *Journal of Financial Economics* 91, 24–37.

Goyal, Amit, and Pedro Santa-Clara, 2003, Idiosyncratic Risk Matters!, *The Journal of Finance* 58, 975–1007.

Goyal, Amit, and Alessio Saretto, 2009, Cross-section of Option Returns and Volatility, *Journal of Financial Economics* 94, 310–326.

Grullon, Gustavo, Evgeny Lyandres, and Alexei Zhdanov, 2012, Real Options, Volatility, and Stock Returns, *Journal of Finance* 67, 1499–1537.

Han, Bing, and Yi Zhou, 2012, Variance Risk Premium and Cross-section of Stock Returns, *Working Paper*.

Hansen, Peter R, and Asger Lunde, 2006, Realized Variance and Market Microstructure Noise, *Journal of Business & Economic Statistics* 24, 127–161.

Herskovic, Bernard, Bryan Kelly, Hanno Lustig, and Stijn Van Nieuwerburgh, 2016, The common factor in idiosyncratic volatility: Quantitative asset pricing implications, *Journal of Financial Economics* 119, 249–283.

Hou, Kewei, and Roger K Loh, 2016, Have we solved the idiosyncratic volatility puzzle?, *Journal of Financial Economics* 121, 167–194.

Jagannathan, Ravi, and Zhenyu Wang, 1996, The Conditional CAPM and the Cross-Section of Expected Returns, *The Journal of Finance* 51, 3.

Kandel, Shmuel, and Robert F. Stambaugh, 1990, Expectations and Volatility of Consumption and Asset Returns, *Review of Financial Studies* 3, 207–232.

Newey, Whitney K, and Kenneth D West, 1987, A Simple, Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica* 55, 703–708.

Officer, R. R., 1973, The Variability of the Market Factor of the New York Stock Exchange, *The Journal of Business* 46, 434—-453.

Pan, Jun, 2002, The jump-risk premia implicit in options: evidence from an integrated time-series study, *Journal of Financial Economics* 63, 3–50.

Pástor, uboš, and Pietro Veronesi, 2009, Technological Revolutions and Stock Prices, *American Economic Review* 99, 1451–1483.

Pyun, Sungjune, 2019, Variance risk in aggregate stock returns and time-varying return predictability, *Journal of Financial Economics* 132, 150–174.

Schwert, G. William, 1989, Why Does Stock Market Volatility Change Over Time?, *The Journal of Finance* 44, 1115–1153.

Segal, Gill, Ivan Shaliastovich, and Amir Yaron, 2015, Good and bad uncertainty: Macroeconomic and financial market implications, *Journal of Financial Economics* 117, 369–397.

Stambaugh, Robert F, Jianfeng Yu, and Yu Yuan, 2015, Arbitrage Asymmetry and the Idiosyncratic Volatility Puzzle, *The Journal of Finance* 70, 1903–1948.

Whitelaw, Robert F., 1994, Time Variations and Covariations in the Expectation and Volatility of Stock Market Returns, *The Journal of Finance* 49, 515–541.

Xing, Yuhang, Xiaoyan Zhang, and Rui Zhao, 2010, What Does the Individual Option Volatility Smirk Tell Us About Future Equity Returns?, *Journal of Financial and Quantitative Analysis* 45, 641–662.

A. Appendix

1. Proof of Result 2

Consider the stock i's ($S_{i,t}$) process given as in the main text, where it can be represented as the sum of the systematic risk and unpriced idiosyncratic risk. That is, let

$$\frac{dS_{i,t}}{S_{i,t}} = a_i dt + dY_{i,t} + \sigma_{idio} dW_{i,t}^{idio},$$

where $dY_{i,t} = \sum_n \beta_{n,i} dF_n$. Further, assume that factor n follows a stochastic process with leverage parameter ρ_n.

$$\begin{aligned} dF_{n,t} &= \mu_{n,t} dt + \sqrt{V_{nt}}(\rho_n dW_{n,t}^v + \sqrt{1-\rho_n^2} dW_{n,t}^o) \\ dV_{n,t} &= \theta_n dt + \sigma_{n,v} dW_{n,t}^v. \end{aligned}$$

The next step is to express the systematic risk component of stock i (Y_i) as functions of other parameters. Denote the conditional expected variation of Y_i at time t by $\bar{\mu}_{i,t} = \sum_n \beta_{n,i} \mu_{n,t}$, and the conditional variance by $\bar{V}_{i,t} = \sum_n \beta_{n,i}^2 V_{n,t}$. From the previous relation, it follows that $d\bar{V}_{i,t} = \sum_n b_i^2 \beta_{n,i}^2 dV_{nt}$. Let $\bar{\theta}_{i,t} = \sum_n \beta_{n,i}^2 \theta_n$ and $\bar{\sigma}_i^2 = \sum_n (\beta_{n,i}^2 \sigma_{n,v})^2$. Then the above equations can be rewritten as,

$$\begin{aligned} dY_{i,t} &= \bar{\mu}_{i,t} dt + \sqrt{\bar{V}_{i,t}}(\bar{\rho}_i dW_{i,t}^v + \sqrt{1-\bar{\rho}_i^2} dW_{i,t}^o) \\ d\bar{V}_{i,t} &= \bar{\theta}_i dt + \bar{\sigma}_i dW_{i,t}^v, \end{aligned}$$

where $\bar{\rho}_i$ measures the correlation between $dY_{i,t}$ and $d\bar{V}_{i,t}$. Following the same steps of Result 1, one can show that

$$E_t[R_{i,t+1}] = \beta_{Y,i,t} \lambda_{Y,i,t} + \lambda_{o,i,t},$$

where $\lambda_{Y,i,t}$ is the price of stock i's systematic variance risk, $\lambda_{o,i,t}$ is the price of risk that comes from orthogonal shocks, and $\beta_{Y,i,t}$ is the slope of a hypothetical regression of stock

i's return on the variance of latent systematic risk dY_i component of stock returns. Under the assumption that idiosyncratic volatility is constant, the first component above ($\beta_{Y,i,t}\lambda_{Y,i,t}$) equals the product of price and quantity of the variance of stock i ($dV_{i,t}$). This can be observed since $\lambda_{v,i,t} = \lambda_{Y,i,t}$ (idiosyncratic variance is not priced)

$$\begin{aligned}\beta_{Y,i,t}\lambda_{Y,i,t} &= Cov(dS_{i,t}/S_{i,t}, d\bar{V}_{i,t})/Var(d\bar{V}_{i,t})\lambda_{Y,i,t} \\ &= \beta_{v,i,t}\lambda_{v,i,t}.\end{aligned}$$

2. Variable Definitions

The following variables, calculated using variables obtained from Compustat North America, are used in this paper.

Market Value of Equity The market value of equity is a proxy for firm size and is computed by multiplying the end-of-year stock price (PRCC_F) with the number of shares outstanding (CSHO).

Book Value of Equity The book value of equity is also a proxy for firm size and is the value of common/ordinary equity (CEQ).

Leverage Leverage is a proxy for firm leverage, corresponding to the moneyness of options. It is the log ratio of liability to the sum of market value of equity (as defined above) and liability. Liability is computed as the sum of debt in current liabilities (DLC) and long-term debt (DLTT). Whether the leverage should positively or negatively affect the risk exposure is uncertain. Higher leverage would imply stronger leverage effect (negative exposure), but at the same time, it would imply higher moneyness (positive exposure).

Present Value of Growth Option (PVGO) The PVGO is the percentage of equity value arising from grown opportunities as to the total market value and follows the methodology of Cao, Simin, and Zhao (2008). The PVGO is estimated from the following steps. First, the return on equity (ROE) is estimated by dividing the operating cash flows by the book value of the non-debt long-term liability. The weighted average of the most recent four years of ROEs, with weights 0.4, 0.3, 0.2, and 0.1, is taken every year so that recent observations are given more weights. Second, projected earnings are obtained by multiplying the average of the ROEs by the non-debt long-term liability. Third, the value of the asset in place is estimated by discounting the projected cash flows. The discount rate is computed using a CAPM model, the one-month risk-free rate and the previous 60-month stock returns. Finally, the PVGO is computed by subtracting the value of the asset in place from the market value of equity.

Profitability Profitability is defined as the earnings before interest and tax (EBIT) divided by total assets (AT).

The following variables are used to construct macroeconomic factors and factor betas considered in the paper. The betas are estimated using a two-variable regression that includes the market factor.

Consumption Growth The construction of consumption growth is defined as the log of the first-order difference in total monthly consumption in non-durables and services in 1992 chained dollars. Consumption betas are estimated using quarterly returns over the 5-year rolling window.

Production Growth Following Chen, Roll, and Ross (1986), production growth is defined as the log difference in the industrial production. I use the seasonally adjusted industrial production index obtained from the Federal Reserve of St. Louis. Production growth is the monthly first-order difference, and the betas are estimated using monthly returns over the 5-year rolling window.

Income Growth Following Jagannathan and Wang (1996), the data on personal income and population are taken from Table 2.2 in the National Income and Product Account of the U.S.A. published by the Bureau of Economic Analysis, U.S. Department of Commerce. Income is defined as personal income excluding dividends. I take the log difference of the quarterly income and estimate the betas using quarterly returns over the 10-year rolling window.

Default Premium Following Chen, Roll, and Ross (1986), default premium is defined as the difference between the Baa and Aaa corporate spreads. I use Moody's seasoned corporate bond yield from the Federal Reserve of St. Louis. I use monthly data and estimate the betas within a 5-year rolling window.

Inflation Inflation is the first order log difference in the consumper price index for the United States excluding foods and energy. It is obtained from the Federal Reserve of St. Louis. The betas are estimated using a 5-year rolling window.

Unemployment Rate The unemployment rate is from the Bureau of Labor Statistics and is seasonally adjusted. I take the first difference of this index and estimate the betas using a 5-year rolling window.

B. Figures and Tables

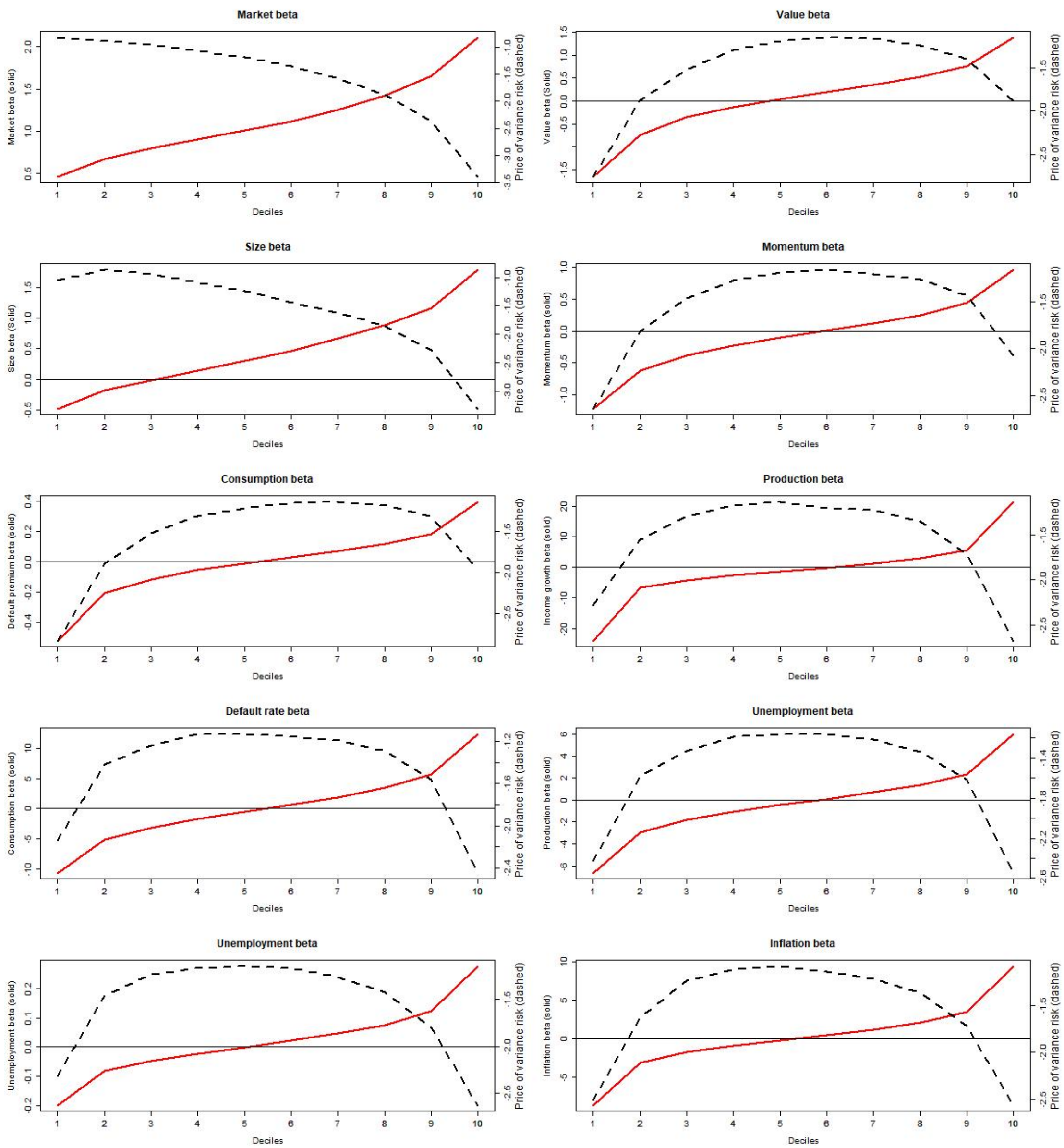

Figure 1. The stock-specific price of variance risk and factor betas

This figure shows the relationship between the price of individual stock variance risk and the betas of various risk factors. Each figure represents a distinct factor. Stocks are first sorted by their factor exposures, decile portfolios are formed from their rankings. Then, the average of the factor betas (in solid lines) and the price of variance risk (in dashed lines) are calculated for each of these portfolios. By construction, the factor betas have a positive slope.

Table I
Summary Statistics

This table provides the summary statistics for several variables of interest. Implied variance (IV) is the average option-implied variance of at-the-money call and put options. Historical realized variance (RV) and bi-power variation (BPV) are estimated using 75-minute intraday returns over the past month. The variance risk exposure ($\beta_{v,i}$) is the slope of the regression

$$R_{i,t} = \beta_{0,i} + \beta_{m,i} R_{m,t} + \beta_{v,i} \left[IV_{i,t} - IV_{i,t-1} \right] + \epsilon_{i,t},$$

where $R_{i,t}$ is the excess return of stock i and $R_{m,t}$ is the value-weighted excess market return. This regressions are estimated using daily data over the 6-month rolling window. The mean, median, and standard deviations for the sample as well as the mean of the entire CRSP database are also provided.

	Entire CRSP	Sample				
	Mean	Mean	Median	St Dev	% Negative	% Positive
IV_i	-	0.018	0.011	0.025	-	100
RV_i	0.022	0.014	0.007	0.021	-	100
BPV_i	0.005	0.003	0.002	0.005	-	100
$RV_i - IV_i$	-	-0.004	-0.003	0.024	0.725	0.275
$BPV_i - IV_i$	-	-0.015	-0.009	0.022	0.980	0.020
$\hat{\beta}_{v,i}$	-	-1.565	-1.155	2.301	0.710	0.290
Market Beta	1.097	1.158	1.100	0.480	0.005	0.995
Market Cap	2.721B	11.50B	2.881B	3.288B	-	100
# of Stocks	4,948	977	928	370	-	-
# of Stock-Month		245,248				

Table II
The Characteristics and Performance of Single-sorted Portfolios

This table summarizes the performance and characteristics of portfolios sorted by the price and quantity of variance risk. The quantity of variance risk is defined as in Table I, and the price of variance risk is the difference between RV and IV ($\lambda_{i,v}$) or the difference between BPV and IV ($\lambda_{i,v}^{BPV}$). Excess returns (Ret.), risk-adjusted (value, size, momentum, profitability, investment) returns (α_6) of the subsequent month, the average of the contemporaneous values of the price and quantity, firm size, and the market beta of each of these portfolios are reported. In Panel B, the values of several additional variables are reported. log (D/E) is the log of the debt to equity ratio. PVGO is constructed following Long, Wald, and Zhang as the present value of market value of equity minus the value of asset in place. Profitability (Prof.) is earnings before interest expense and tax divided by total assets.

Panel A. Portfolios sorted by the price of individual stock variance risk ($\lambda_{v,i}$)

	Ret.	α_6	$\lambda_{v,i}$	$\hat{\beta}_{v,i}$	Size	Market Beta
Quintile 1	0.98	0.48	−0.022	−1.370	2.79B	1.104
	(1.60)	(1.64)				
Quintile 2	1.08	0.51	−0.008	−2.029	5.71B	1.085
	(2.34)	(3.08)				
Quintile 3	0.84	0.19	−0.004	−2.610	9.17B	1.070
	(2.65)	(2.36)				
Quintile 4	0.57	0.00	−0.002	−3.206	16.02B	1.060
	(2.05)	(−0.03)				
Quintile 5	0.23	−0.25	0.002	−3.907	26.42B	0.947
	(0.63)	(−2.43)				
Q5-Q1	−0.75*	−0.73**	0.024	−2.537	23.63B	−0.157
	(−1.74)	(−2.19)				

Panel B. Portfolios sorted by the individual stock variance risk exposure ($\hat{\beta}_{v,i}$)

	Ret.	α_6	$\lambda_{v,i}$	$\lambda_{i,v}^{BPV}$	$\hat{\beta}_{v,i}$	Size	log (D/E)	PVGO	Prof.
Quintile 1	0.83	0.08	−0.001	−0.009	−5.044	24.34B	−2.01	0.128	0.135
	(3.14)	(0.73)							
Quintile 2	0.57	0.15	−0.002	−0.013	−2.471	13.38B	−1.63	0.158	0.121
	(1.50)	(1.63)							
Quintile 3	0.69	0.25	−0.003	−0.017	−1.330	8.74B	−1.48	0.148	0.108
	(1.79)	(1.63)							
Quintile 4	0.65	0.03	−0.003	−0.021	−0.048	5.97B	−1.48	0.176	0.983
	(1.76)	(0.24)							
Quintile 5	0.72	−0.01	−0.001	−0.017	0.098	5.20B	−1.45	0.156	0.981
	(2.39)	(−0.08)							
Q5-Q1	−0.11	−0.09	0.000	−0.008	5.142	−19.14B	0.56	0.028	0.846
	(−0.53)	(−0.42)							

Table III
The Performance of Price-Quantity of Variance Risk Double-sorted Portfolios

This table summarizes the value-weighted returns of the price and quantity of stock variance risk (independent) double-sorted portfolios. In Panel A, the price of variance risk ($\lambda_{v,i}$) is defined as the difference between realized variance (RV) and option-implied variance (IV). In Panel B, it ($\lambda_{v,i}^{BPV}$) is defined as the difference between bi-power variation (BPV) and IV. The quantity of variance risk is the slope of the regression,

$$R_{i,t} = \beta_{0,i} + \beta_{m,i} R_{m,t} + \beta_{v,i} \left[IV_{i,t} - IV_{i,t-1} \right] + \epsilon_{i,t},$$

where $R_{i,t}$ is the excess return of stock i, and $R_{m,t}$ is the value-weighted excess market return. The regression is estimated using daily data over the six-month rolling window. In Panel C, the price is defined as the difference between the GARCH forecast of stock variance and IV. The quantity is estimated using innovations of the GARCH forecasts, instead of IV. Excess returns (Ret.), risk-adjusted (value, size, growth, profitability, investment) returns (α_6) of the subsequent month are reported.

Panel A. RV-based price of variance risk ($\lambda_{v,i} = RV_i - IV_i$)

	Variance risk exposure ($\widehat{\beta_{v,i}}$)									
$\lambda_{v,i}$	Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6
Q1 (Negative)	1.50	1.23	0.85	0.82	1.13	0.80	0.59	0.23	−0.91*	−1.00*
	(2.31)	(2.94)	(1.18)	(2.27)	(1.82)	(1.82)	(1.00)	(0.73)	(−1.82)	(−1.87)
Q2	1.18	0.45	0.48	0.09	0.95	0.51	0.88	0.16	−0.30	−0.29
	(3.10)	(2.71)	(1.16)	(0.53)	(2.03)	(2.41)	(2.28)	(0.99)	(−1.23)	(−1.31)
Q3	0.80	0.45	0.48	0.09	0.95	0.51	0.88	0.16	−0.30	−0.29
	(2.94)	(2.71)	(1.16)	(0.53)	(2.03)	(2.41)	(2.28)	(0.99)	(−1.23)	(−1.31)
Q4 (Positive)	0.15	−0.32	0.24	−0.07	0.39	−0.02	0.49	−0.23	0.34	0.09
	(0.45)	(−2.18)	(0.67)	(−0.47)	(1.03)	(−0.11)	(1.58)	(−0.93)	(1.36)	(0.29)
Q4-Q1	−1.35**	−1.55***	−0.61	−0.89**	−0.74	−0.82*	−0.10	−0.46	1.25**	1.09**
	(−2.36)	(−3.20)	(−1.16)	(−2.45)	(−1.62)	(−1.89)	(−0.22)	(−1.44)	(2.47)	(2.24)

Panel B. BPV-based price of variance risk ($\lambda_{v,i}^{BPV} = BPV_i - IV_i$)

	Variance risk exposure ($\widehat{\beta_{v,i}}$)									
$\lambda_{v,i}^{BPV}$	Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6
Q1 (Negative)	1.27	1.25	0.71	0.96	0.83	0.53	0.16	−0.31	−1.11**	−1.56**
	(2.07)	(2.58)	(0.90)	(2.16)	(1.08)	(1.17)	(0.25)	(−0.69)	(−2.02)	(−3.02)
Q2	1.07	0.46	0.68	0.44	0.97	0.56	0.76	0.02	−0.31	−0.44
	(2.40)	(2.23)	(1.31)	(2.07)	(1.79)	(2.28)	(1.80)	(0.09)	(−0.91)	(−1.36)
Q3	0.96	0.46	0.68	0.44	0.97	0.56	0.76	0.02	−0.31	−0.44
	(2.54)	(2.23)	(1.31)	(2.07)	(1.79)	(2.28)	(1.80)	(0.09)	(−0.91)	(−1.36)
Q4 (Positive)	0.68	−0.05	0.31	−0.32	0.49	−0.23	0.64	0.00	−0.04	0.05
	(2.85)	(−0.34)	(0.97)	(−2.02)	(1.71)	(−1.74)	(2.56)	(0.03)	(−0.23)	(0.29)
Q4-Q1	−0.59	−1.29**	−0.40	−1.28**	−0.34	−0.76	0.48	0.31	1.07*	1.61**
	(−1.14)	(−2.29)	(−0.58)	(−2.39)	(−0.51)	(−1.57)	(0.87)	(0.64)	(1.91)	(2.92)

Panel C. Parametric Modeling using GARCH (1,1)

	Variance risk exposure ($\widehat{\beta_{v,i}^{G}}$)									
$\lambda_{v,i}^{G}$	Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6
Q1 (Negative)	1.78	1.30	0.59	0.19	1.13	0.46	1.26	0.59	−0.51*	−0.71***
	(4.45)	(4.41)	(1.07)	(0.56)	(2.18)	(1.56)	(3.04)	(2.30)	(−1.76)	(−2.71)
Q2	0.91	0.18	1.20	0.36	0.69	−0.21	0.75	0.02	−0.16	−0.16
	(2.91)	(1.24)	(3.01)	(1.40)	(2.05)	(−1.09)	(2.50)	(0.14)	(−0.71)	(−0.81)
Q3	0.73	0.00	0.78	0.27	0.67	0.02	0.43	−0.26	−0.30*	−0.26*
	(2.41)	(−0.01)	(1.93)	(1.19)	(1.79)	(0.13)	(1.37)	(−2.16)	(−1.94)	(−1.76)
Q4 (Positive)	−0.09	−0.60	0.03	−0.33	0.22	−0.09	0.55	0.23	0.64**	0.83***
	(−0.20)	(−2.99)	(0.05)	(−1.05)	(0.47)	(−0.39)	(1.35)	(1.44)	(2.29)	(3.28)
Q4-Q1	−1.86***	−1.90***	−0.56	−0.52	−0.90*	−0.55	−0.71*	−0.35	1.15***	1.55***
	(−4.89)	(−5.34)	(−1.22)	(−1.10)	(−1.82)	(−1.51)	(−1.93)	(−1.13)	(3.10)	(4.39)

Table IV
Alternative Specifications of the Performance of Double-sorted Portfolios

This table shows the performance of the double sorted portfolios of Table III under several alternative specifications. In Panel A, the portfolios are first sorted by the price and then by the quantity within each group. In panel B, the portfolios are first sorted by the quantity and then by the price. In Panel C, weighted least squares (WLS) is used to estimate the quantity. Finally, in panel D, the quantity is estimated using a single–variable linear regression of

$$R_{i,t} = \beta_{0,i} + \beta^1_{v,i}\left[IV_{i,t} - IV_{i,t-1}\right] + \epsilon_{i,t}$$

A. Price-quantity dependent sort

$\lambda_{v,i}$	Variance risk exposure ($\widehat{\beta_{v,i}}$)									
	Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6
Q1 (Negative)	1.27	1.18	0.96	0.87	1.05	0.34	0.66	0.18	−0.61	−1.00**
	(1.99)	(3.98)	(1.39)	(2.55)	(1.59)	(0.75)	(1.19)	(0.52)	(−1.49)	(−2.42)
Q2	1.28	0.44	0.84	0.25	0.97	0.76	0.79	0.17	−0.49*	−0.27
	(3.74)	(2.52)	(1.85)	(1.28)	(1.94)	(2.87)	(2.11)	(1.02)	(−1.94)	(−1.19)
Q3	0.80	0.44	0.84	0.25	0.97	0.76	0.79	0.17	−0.49*	−0.27
	(2.85)	(2.52)	(1.85)	(1.28)	(1.94)	(2.87)	(2.11)	(1.02)	(−1.94)	(−1.19)
Q4 (Positive)	0.31	−0.15	0.16	−0.16	0.33	−0.10	0.51	−0.28	0.20	−0.13
	(1.08)	(−1.20)	(0.41)	(−1.28)	(0.83)	(−0.61)	(1.51)	(−1.22)	(0.78)	(−0.49)
Q4-Q1	−0.96*	−1.33***	−0.81	−1.04***	−0.72	−0.44	−0.16	−0.46	0.80**	0.86**
	(−1.83)	(−3.69)	(−1.45)	(−2.92)	(−1.44)	(−0.98)	(−0.38)	(−1.24)	(1.99)	(2.00)

B. Quantity-price dependent sort

$\lambda_{v,i}$	Variance risk exposure ($\widehat{\beta_{v,i}}$)									
	Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6
Q1 (Negative)	1.41	0.70	0.95	1.00	0.83	0.44	0.44	−0.13	−0.97**	−0.83**
	(3.26)	(3.50)	(1.29)	(2.69)	(1.23)	(0.86)	(0.74)	(−0.35)	(−2.43)	(−1.99)
Q2	0.99	0.27	0.99	0.50	1.03	0.52	0.75	0.21	−0.24	−0.06
	(3.03)	(1.96)	(2.18)	(2.59)	(1.76)	(1.99)	(1.85)	(1.09)	(−0.91)	(−0.22)
Q3	0.78	0.27	0.99	0.50	1.03	0.52	0.75	0.21	−0.24	−0.06
	(2.83)	(1.96)	(2.18)	(2.59)	(1.76)	(1.99)	(1.85)	(1.09)	(−0.91)	(−0.22)
Q4 (Positive)	0.35	−0.22	0.30	−0.09	0.19	−0.15	0.51	−0.28	0.16	−0.06
	(1.15)	(−1.61)	(0.78)	(−0.63)	(0.47)	(−1.03)	(1.59)	(−1.18)	(0.59)	(−0.23)
Q4-Q1	−1.06***	−0.91***	−0.66	−1.09***	−0.64	−0.59	0.06	−0.15	1.13**	0.76*
	(−3.08)	(−3.55)	(−1.21)	(−2.76)	(−1.20)	(−1.15)	(0.14)	(−0.38)	(2.45)	(1.90)

C. Weighted least squares

$\lambda_{v,i}$	Variance risk exposure ($\widehat{\beta^{WLS}_{v,i}}$)									
	Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6
Q1 (Negative)	1.60	1.27	0.87	0.91	1.07	0.62	0.52	0.03	−1.07**	−1.24***
	(3.06)	(3.60)	(1.14)	(2.45)	(1.62)	(1.59)	(0.95)	(0.09)	(−2.39)	(−2.89)
Q2	1.28	0.45	0.94	0.36	0.92	0.67	0.82	0.16	−0.46*	−0.29
	(3.60)	(2.61)	(2.16)	(1.87)	(1.86)	(2.59)	(2.17)	(0.92)	(−1.75)	(−1.21)
Q3	0.84	0.45	0.94	0.36	0.92	0.67	0.82	0.16	−0.46*	−0.29
	(2.92)	(2.61)	(2.16)	(1.87)	(1.86)	(2.59)	(2.17)	(0.92)	(−1.75)	(−1.21)
Q4 (Positive)	0.27	−0.23	0.31	−0.01	0.31	−0.15	0.52	−0.29	0.24	−0.06
	(0.92)	(−1.85)	(0.82)	(−0.04)	(0.75)	(−0.95)	(1.55)	(−1.28)	(0.95)	(−0.23)
Q4-Q1	−1.32***	−1.50***	−0.56	−0.92**	−0.75	−0.76*	−0.01	−0.32	1.32***	1.17**
	(−2.90)	(−3.67)	(−1.01)	(−2.38)	(−1.46)	(−1.88)	(−0.02)	(−0.87)	(2.79)	(2.28)

D. Single factor estimation of the quantity

$\lambda_{v,i}$	Variance risk exposure ($\widehat{\beta^{1}_{v,i}}$)									
	Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6
Q1 (Negative)	1.72	1.40	0.79	0.78	1.01	0.75	0.49	−0.15	−1.23**	−1.55***
	(2.79)	(3.06)	(1.20)	(2.57)	(1.55)	(2.59)	(0.90)	(−0.35)	(−2.42)	(−3.09)
Q2	1.17	0.45	0.99	0.52	0.82	0.40	0.95	0.22	−0.22	−0.23
	(3.35)	(2.84)	(2.15)	(2.54)	(1.74)	(1.81)	(2.26)	(1.14)	(−0.82)	(−1.00)
Q3	0.77	−0.04	0.50	−0.07	0.73	0.11	0.51	−0.07	−0.26	−0.03
	(2.65)	(−0.32)	(1.55)	(−0.53)	(2.10)	(0.73)	(1.66)	(−0.48)	(−1.09)	(−0.15)
Q4 (Positive)	0.26	−0.22	0.27	−0.21	0.40	−0.08	0.86	0.06	0.59**	0.28
	(0.90)	(−2.09)	(0.76)	(−1.20)	(0.98)	(−0.48)	(2.44)	(0.23)	(2.20)	(1.09)
Q4-Q1	−1.46***	−1.62***	−0.51	−1.00***	−0.61	−0.82***	0.37	0.21	1.82***	1.83***
	(−2.68)	(−3.23)	(−0.95)	(−2.82)	(−1.23)	(−2.65)	(0.96)	(0.49)	(3.44)	(3.34)

Table V
The Performance of Double-sorted Portfolios Controlling for Market Variance Risk

This table summarizes the performance of the price and quantity of variance risk double-sorted portfolios after controlling for the stock's exposure to market variance risk. In Panel A, the sorting is based on the price and quantity of idiosyncratic variance risk. The price of idiosyncratic variance risk ($\lambda_{i,e,t}$) of stock i at time t is estimated as

$$\lambda_{e,i,t} = \lambda_{v,i,t} - \hat{\beta}_{vx,i}\lambda_{x,t},$$

where $\lambda_{x,t}$ is the price of market variance risk and $\hat{\beta}_{vx,i}$ is the slope of individual stock variance risk regressed on market variance risk.

$$IV_{i,t} - IV_{i,t-1} = \beta_{vx,0} + \beta_{vx,i}(VIX_t^2 - VIX_{t-1}^2) + e_t,$$

where $IV_{i,t}$ is the option implied variance of stock i, and VIX^2 is the square of the market volatility index. The quantity ($\beta_{e,i}$) is estimated from

$$R_{i,t} = \alpha + \beta_{m,i}R_{m,t} + \beta_{e,i}\hat{e}_t + \epsilon_t$$

In Panel B, the price and quantity of risk is estimated as in the main specification, but the returns (α_7) are adjusted by the market variance risk factor (FVIX) of Ang, Hodrick, Xing, and Zhang (2006) in addition to the six factors of Fama and French (2015).

A. Sorted by price-quantity of idiosyncratic variance risk

	Variance risk exposure ($\widehat{\beta_{v,i}}$)									
$\lambda_{v,i}$	Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6
Q1 (Negative)	1.43	1.25	0.89	0.86	1.11	0.67	0.63	0.18	−0.80*	−1.08**
	(2.69)	(3.46)	(1.17)	(2.41)	(1.70)	(1.74)	(1.16)	(0.48)	(−1.81)	(−2.36)
Q2	1.32	0.52	0.88	0.31	0.82	0.48	0.86	0.18	−0.45*	−0.35
	(3.72)	(3.17)	(1.95)	(1.43)	(1.71)	(1.96)	(2.31)	(1.05)	(−1.79)	(−1.60)
Q3	0.82	0.04	0.50	−0.13	0.48	−0.20	0.72	0.10	−0.10	0.06
	(2.80)	(0.29)	(1.54)	(−1.22)	(1.42)	(−1.56)	(2.40)	(0.70)	(−0.43)	(0.29)
Q4 (Positive)	0.28	−0.21	0.23	−0.06	0.42	−0.08	0.61	−0.22	0.33	−0.01
	(0.93)	(−1.77)	(0.60)	(−0.41)	(1.05)	(−0.53)	(1.93)	(−0.99)	(1.40)	(−0.02)
Q4-Q1	−1.47***	−1.47***	−0.66	−0.92**	−0.69	−0.75*	−0.03	−0.40	1.13**	1.07*
	(−2.51)	(−3.49)	(−1.19)	(−2.51)	(−1.32)	(−1.82)	(−0.07)	(−1.02)	(2.45)	(1.95)

B. Market variance risk adjusted returns

	Variance risk exposure ($\widehat{\beta_{v,i}}$)				
$\lambda_{v,i}$	Q1 (Negative)	Q2	Q3	Q4 (Positive)	Q4-Q1
	α_7	α_7	α_7	α_7	α_7
Q1 (Negative)	1.04	0.66	0.50	−0.27	−1.31***
	(2.98)	(1.96)	(1.43)	(−0.92)	(−2.97)
Q2	0.58	0.31	0.47	0.03	−0.54**
	(3.43)	(1.70)	(1.93)	(0.19)	(−2.25)
Q3	0.23	−0.06	−0.39	0.13	−0.10
	(1.90)	(−0.50)	(−2.54)	(0.92)	(−0.54)
Q4 (Positive)	−0.08	−0.15	−0.20	−0.22	−0.15
	(−0.68)	(−0.90)	(−1.34)	(−1.09)	(−0.65)
Q4-Q1	−1.12***	−0.80**	−0.71*	0.05	1.16**
	(−2.90)	(−2.19)	(−1.73)	(0.13)	(2.22)

Table VI
The Quantity of Variance Risk, Factor Betas, and Factor Return-Variance Covariance

This table summarizes the output of the following two-stage regression: The first-stage is the cross-sectional regression

$$\hat{\beta}^{G}_{v,i,t} = \delta_{0,t} + \sum_{j} \delta_{j,t}\hat{\beta}_{j,i,t} + \epsilon_{i,t},$$

where $\hat{\beta}^{G}_{v,i,t}$ is the quantity of variance risk estimated using GARCH (1,1) over three months, $\hat{\beta}_{j,i,t}$ is the beta of stock i on the jth factor, estimated quarterly. The second stage is a time-series regression of

$$\hat{\delta}_{j,t} = \gamma_{0,j} + \gamma_j \widehat{Lev}_{j,t} + e_{j,t},$$

where $\widehat{Lev}_{j,t}$ are the factors' exposures on their variance shocks estimated over three month. The slope coefficient (γ_j) of the final-stage time-series regression is reported, after multiplying by 10,000, along with the Newey-West t-statistics. Panel A shows the result when the market factor is included to estimate the quantity of variance risk, whereas in Panel B, for $\widehat{\beta^{1,G}_{v,i,t}}$, the market factor is excluded.

A. Two-factor quantity estimation ($\widehat{\beta^{G}_{v,i,t}}$)

Factor	$\hat{\gamma}$	Factor	$\hat{\gamma}$
Market	−4.22	HML	6.44*
	(−0.41)		(1.90)
SMB	2.49	UMD	10.30**
	(0.58)		(2.50)
CMA	4.79	RMW	7.38
	(1.50)		(1.49)

B. One-factor quantity estimation ($\widehat{\beta^{1,G}_{v,i,t}}$)

Factor	$\hat{\gamma}$	Factor	$\hat{\gamma}$
Market	15.01	HML	4.65*
	(1.31)		(1.70)
SMB	1.73	UMD	13.31***
	(0.38)		(2.83)
CMA	3.80	RMW	6.24
	(1.24)		(1.26)

Table VII
Determinants of Stock Variance and the Price of Variance Risk

This table shows the results of the contemporaneous cross-sectional regressions of

$$\text{Dep}_{i,t} = c_0 + \mathbf{c}'\mathbf{B}_t + \epsilon_{i,t},$$

where Dep is either the option-implied variance of stocks (Panels A and B) or the price of the stock variance risk (Panels C and D), $\mathbf{B}$ is a vector of the squared of betas of some factor model, estimated using six months of data. The regressions are estimated semi-annually in June and December. The averages of the coefficient, as well as the Fama-MacBeth standard errors, are summarized. Panels A and C use macro-economic factors (consumption growth, production growth, unemployment rate, default premium, income growth, and inflation) as described in the main text, while Panels B and D use the factors of Fama and French (2015) and momentum. Both the price and the option-implied variances are multiplied by 100 to make Panel B and D readable. The beta squares are scaled so that the coefficients range between the scale of 0.01–10.

Panel A. Option-implied variance and macro factor betas

	Dep = Option-implied Variance			
β^2_M				0.26***
				(6.20)
$\beta^2_{\text{Consumption}}$	0.25***		0.03	
	(5.31)		(1.08)	
$\beta^2_{\text{Production}}$		1.56***	0.33**	
		(6.00)	(2.26)	
$\beta^2_{\text{Unemployment}}$			3.06***	3.30***
			(2.79)	(3.00)
$\beta^2_{\text{Inflation}}$			18.23***	18.02***
			(5.91)	(6.08)
β^2_{Default}			2.67**	2.65**
			(2.35)	(2.67)
$\beta^2_{\text{Inc Growth}}$			0.27***	0.27***
			(4.57)	(5.40)
# of Obs.	42	42	42	42

Panel B. Option-implied variance and factor betas

	Dep = Option-implied Variance		
β^2_M	0.12**	0.15***	0.12***
	(2.36)	(3.17)	(2.84)
β^2_{SMB}	0.50***	0.50***	0.46***
	(8.61)	(8.09)	(8.84)
β^2_{HML}	0.21***	0.11***	0.10***
	(7.47)	(4.78)	(4.09)
β^2_{UMD}	0.72***		0.57***
	(3.31)		(3.27)
β^2_{CMA}		0.09***	0.08***
		(6.11)	(5.40)
β^2_{RMW}		0.24***	0.22***
		(3.72)	(3.91)
# of Obs.	42	42	42

Panel C. The price of variance risk and macro factor betas

	Dep = $\lambda_{v,i,t}$			
β^2_M				−0.06**
				(−2.54)
$\beta^2_{\text{Consumption}}$	−0.10***		0.01	
	(−3.95)		(0.58)	
$\beta^2_{\text{Production}}$		−0.73***	−0.03	
		(−5.12)	(−0.31)	
$\beta^2_{\text{Unemployment}}$			−1.80**	−1.79**
			(−2.63)	(−2.57)
$\beta^2_{\text{Inflation}}$			−9.72***	−9.62***
			(−4.28)	(−4.26)
β^2_{Default}			−1.23**	−1.34**
			(−2.18)	(−2.51)
$\beta^2_{\text{Inc Growth}}$			−0.14***	−0.14***
			(−2.96)	(−3.13)
# of Obs.	42	42	42	42

Panel D. The price of variance risk and factor betas

	Dep = $\lambda_{v,i,t}$		
β^2_M	0.01	−0.01	0.01
	(0.30)	(−0.31)	(0.29)
β^2_{SMB}	−0.25***	−0.26***	−0.23***
	(−5.55)	(−5.48)	(−5.51)
β^2_{HML}	−0.09***	−0.05**	−0.04*
	(−5.04)	(−2.59)	(−1.93)
β^2_{UMD}	−0.44***		−0.35***
	(−2.85)		(−2.83)
β^2_{CMA}		−0.04***	−0.04***
		(−4.26)	(−3.95)
β^2_{RMW}		−0.13***	−0.12***
		(−2.94)	(−3.11)
# of Obs.	42	42	42

Table VIII
The Performance of Double-sorted Portfolios
Extension to Non-optionable Stocks

This table shows the performance of double-sorted portfolios using predicted price and quantity of variance risk as sorting variables. As a first stage, using only stocks that have options traded, the option-implied variance is fitted from a cross-sectional regression using the square of betas. Then, using regression coefficients implied variance is extrapolated for all stocks. The predicted price of variance risk is the difference between the GARCH(1,1)-based variance forecast and the predicted option-implied variance. The quantity of variance risk is also estimated using a GARCH model. Panel A and B includes all stocks, whereas Panel C and D exclude stocks that have options traded. Panel A and C shows the result when macroeconomic factors are used to extrapolate the option-implied variance, and Panel B and D shows when Fama and French (2015) factors are used.

Panel A. Implied Variance Estimated from Macro Factors - All stocks

	Variance risk exposure ($\widehat{\beta^G_{v,i}}$)									
$\hat{\lambda}^P_{v,i}$	Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6
Q1 (Negative)	0.74	0.14	0.65	0.00	0.78	−0.06	0.44	−0.27	−0.30*	−0.41***
	(2.67)	(1.18)	(1.97)	(0.02)	(2.57)	(−0.41)	(1.69)	(−2.62)	(−1.92)	(−2.86)
Q2	0.72	0.04	0.79	−0.02	0.65	−0.03	0.53	−0.11	−0.18	−0.15
	(2.45)	(0.31)	(2.07)	(−0.13)	(2.01)	(−0.23)	(1.76)	(−1.09)	(−1.21)	(−1.04)
Q3	0.72	0.04	0.79	−0.02	0.65	−0.03	0.53	−0.11	−0.18	−0.15
	(1.79)	(0.31)	(2.07)	(−0.13)	(2.01)	(−0.23)	(1.76)	(−1.09)	(−1.21)	(−1.04)
Q4 (Positive)	0.34	0.04	0.21	0.20	0.03	−0.25	0.90	0.49	0.56	0.45
	(0.57)	(0.14)	(0.31)	(0.71)	(0.05)	(−0.93)	(1.56)	(1.74)	(1.40)	(1.16)
Q4-Q1	−0.40	−0.10	−0.43	0.20	−0.75*	−0.19	0.46	0.77**	0.86**	0.87**
	(−0.95)	(−0.36)	(−0.88)	(0.62)	(−1.71)	(−0.66)	(0.99)	(2.47)	(2.20)	(2.14)

Panel B. Implied Variance Estimated from Fama-French Factors -All Stocks

	Variance risk exposure ($\widehat{\beta^G_{v,i}}$)									
$\hat{\lambda}^P_{v,i}$	Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6
Q1 (Negative)	0.72	−0.06	0.82	0.08	0.78	−0.02	0.36	−0.20	−0.35	−0.13
	(2.51)	(−0.39)	(2.60)	(0.40)	(2.42)	(−0.12)	(1.31)	(−1.55)	(−1.47)	(−0.60)
Q2	0.83	0.17	1.00	0.54	0.55	0.02	0.51	−0.17	−0.32*	−0.34**
	(2.74)	(1.41)	(2.89)	(2.67)	(1.84)	(0.12)	(1.85)	(−1.90)	(−1.92)	(−2.36)
Q3	0.44	0.17	1.00	0.54	0.55	0.02	0.51	−0.17	−0.32*	−0.34**
	(1.02)	(1.41)	(2.89)	(2.67)	(1.84)	(0.12)	(1.85)	(−1.90)	(−1.92)	(−2.36)
Q4 (Positive)	−0.30	−0.58	−0.37	−0.43	−0.26	−0.27	0.50	0.43	0.79*	1.01**
	(−0.47)	(−1.82)	(−0.59)	(−1.59)	(−0.40)	(−1.17)	(0.83)	(1.46)	(1.92)	(2.39)
Q4-Q1	−1.02*	−0.51	−1.18**	−0.50	−1.04**	−0.25	0.13	0.63*	1.15**	1.14**
	(−1.93)	(−1.37)	(−2.49)	(−1.52)	(−2.18)	(−0.86)	(0.29)	(1.84)	(2.40)	(2.41)

Panel C. Implied Variance Estimated from Macro Factors - Non-optionable Stocks

	Variance risk exposure ($\widehat{\beta^G_{v,i}}$)									
$\hat{\lambda}^P_{v,i}$	Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6
Q1 (Negative)	0.76	0.10	0.65	0.06	0.99	0.17	0.51	−0.07	−0.25	−0.17
	(2.54)	(0.67)	(1.74)	(0.29)	(3.17)	(0.97)	(1.93)	(−0.60)	(−1.39)	(−0.96)
Q2	0.84	0.14	0.74	−0.09	0.79	0.26	0.41	−0.30	−0.43**	−0.44***
	(2.50)	(0.92)	(1.72)	(−0.43)	(2.16)	(1.24)	(1.18)	(−2.57)	(−2.45)	(−2.68)
Q3	0.82	0.14	0.74	−0.09	0.79	0.26	0.41	−0.30	−0.43**	−0.44***
	(1.58)	(0.92)	(1.72)	(−0.43)	(2.16)	(1.24)	(1.18)	(−2.57)	(−2.45)	(−2.68)
Q4 (Positive)	0.01	−0.53	0.54	0.21	−0.32	−0.73	0.61	0.15	0.61	0.67
	(0.01)	(−1.56)	(0.71)	(0.42)	(−0.43)	(−1.92)	(1.02)	(0.46)	(1.37)	(1.39)
Q4-Q1	−0.75	−0.62*	−0.11	0.15	−1.31**	−0.90**	0.10	0.22	0.85*	0.84*
	(−1.50)	(−1.77)	(−0.18)	(0.29)	(−2.16)	(−1.98)	(0.20)	(0.67)	(1.88)	(1.80)

Panel D. Implied Variance Estimated from Fama-French Factors - Non-optionable Stocks

	Variance risk exposure ($\widehat{\beta^G_{v,i}}$)									
$\hat{\lambda}^P_{v,i}$	Q1 (Negative)		Q2		Q3		Q4 (Positive)		Q4-Q1	
	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6	Ret.	α_6
Q1 (Negative)	0.71	−0.04	0.72	0.01	0.85	−0.01	0.41	−0.16	−0.30	−0.12
	(2.40)	(−0.27)	(1.96)	(0.06)	(2.28)	(−0.04)	(1.34)	(−1.40)	(−1.33)	(−0.66)
Q2	0.74	0.14	0.65	0.14	0.56	−0.09	0.56	−0.06	−0.18	−0.21
	(2.56)	(1.43)	(1.93)	(0.77)	(1.75)	(−0.60)	(2.13)	(−0.72)	(−1.24)	(−1.60)
Q3	0.63	0.14	0.65	0.14	0.56	−0.09	0.56	−0.06	−0.18	−0.21
	(1.48)	(1.43)	(1.93)	(0.77)	(1.75)	(−0.60)	(2.13)	(−0.72)	(−1.24)	(−1.60)
Q4 (Positive)	−0.03	−0.27	−0.27	−0.28	−0.18	−0.23	0.61	0.28	0.64	0.56
	(−0.05)	(−1.04)	(−0.40)	(−1.01)	(−0.29)	(−0.92)	(1.05)	(0.93)	(1.57)	(1.40)
Q4-Q1	−0.74	−0.24	−0.99*	−0.29	−1.03**	−0.22	0.20	0.44	0.94**	0.68
	(−1.63)	(−0.76)	(−1.93)	(−0.85)	(−2.44)	(−0.70)	(0.45)	(1.30)	(2.14)	(1.60)

Table IX
Cross-sectional Regressions of Monthly Stock Returns

This table summarizes the results of the monthly cross-sectional regressions of

$$R_{i,t+1} = b_0 + b_1 \hat{\lambda}^P_{v,i,t} + b_2 \hat{\beta}^G_{v,i,t} + \mathbf{c}'\mathbf{Control}_{i,t} + \epsilon_{t+1},$$

where $\hat{\lambda}^P_{v,i,t}$ is the predicted price of variance risk extrapolated from optionable stocks, $\hat{\beta}^P_{v,i,t}$ is the quantity of variance risk estimated from a GARCH (1,1) model, and $\mathbf{Control}_{i,t}$ is a vector of control variables. $\hat{\beta}_{M,t}$ is the market beta, Mom_t is the performance of the stock from month $t-12$ to $t-1$, B/M is the log of the book-to-market ratio, Size is the log of the market capitalization, CIV is the common factors of idiosyncratic volatility of Herskovic, Kelly, Lustig, and Van Nieuwerburgh (2016), Ivol is the idiosyncratic volatility, and Growth and Profitability are defined as in the appendix. The time-series average of the coefficients along with the Newey-West adjusted Fama-MacBeth standard errors are reported.

A. Implied Variance Estimated from Macro Factors

	Dep = $R_{i,t+1}$					
$\hat{\lambda}^P_{v,i,t}$	−3.291∗∗∗	−3.222∗∗∗		−2.088∗∗		−2.482∗∗∗
	(−2.66)	(−3.10)		(−2.48)		(−2.63)
$\hat{\beta}^G_{v,i,t}$	−0.015∗∗	−0.014∗∗		−0.013∗∗∗		−0.011∗∗
	(−2.47)	(−2.49)		(−2.64)		(−2.40)
$\hat{\beta}_{M,t}$		−0.13			−0.13	−0.13
		(−0.66)			(−0.73)	(−0.73)
Mom_t		0.24			0.24	0.245∗
		(1.25)			(1.61)	(1.72)
B/M_t		0.16			0.13	0.12
		(1.50)			(1.40)	(1.32)
Size_t		−0.06			−0.06	−0.07
		(−1.02)			(−1.17)	(−1.46)
CIV_t			−0.008∗	−0.008∗	−0.01	−0.01
			(−1.92)	(−1.85)	(−1.28)	(−1.26)
Ivol_t			−1.77	0.70		
			(−0.20)	(0.08)		
Growth_t					−0.567∗∗∗	−0.565∗∗∗
					(−3.77)	(−3.74)
Profitability_t					1.319∗∗	1.260∗∗
					(2.38)	(2.39)

B. Implied Variance Estimated from Fama-French Factors

	Dep = $R_{i,t+1}$					
$\hat{\lambda}^P_{v,i,t}$	−2.945∗∗	−3.259∗∗∗		−2.004∗∗		−2.493∗∗
	(−2.54)	(−3.06)		(−2.29)		(−2.48)
$\hat{\beta}^G_{v,i,t}$	−0.015∗∗	−0.013∗∗		−0.012∗∗		−0.011∗∗
	(−2.26)	(−2.23)		(−2.49)		(−2.14)
$\hat{\beta}_{M,t}$		−0.15			−0.16	−0.16
		(−0.78)			(−0.85)	(−0.84)
Mom_t		0.24			0.24	0.237∗
		(1.32)			(1.61)	(1.69)
B/M_t		0.19			0.15	0.14
		(1.63)			(1.56)	(1.49)
Size_t		−0.07			−0.07	−0.08
		(−0.99)			(−1.16)	(−1.47)
CIV_t			−0.009∗∗	−0.009∗∗	−0.01	−0.01
			(−2.03)	(−2.02)	(−1.30)	(−1.31)
Ivol_t			−4.40	−1.79		
			(−0.51)	(−0.21)		
Growth_t					−0.623∗∗∗	−0.620∗∗∗
					(−4.24)	(−4.23)
Profitability_t					1.414∗∗∗	1.363∗∗∗
					(2.82)	(2.83)